

# On some papers about varieties and quasivarieties of centrally nilpotent loops

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## Abstract

This paper specifies and often corrects the gross mistakes, incorrect, erroneous and senseless statements that can be found practically in all published works of V. Ursu and A. Covalschi. It also specifies numerous cases of plagiarism.

**Key words:** commutative Moufang loop, Moufang loop,  $A$ -loop, nilpotent loop, nilpotent loop of class 2, variety, quasivariety, basis of identities, basis of quasiidentities, separable loop, lattice, covering.

**Mathematics of subject classification:** 17D05, 20N05.

## 1 Preliminaries

This paper analyzes all works of V. Ursu and A. Covalschi. Ideologically, this paper is similar to papers [44], [45]. All analyzed works of the given paper are not scientific, their results are erroneous, similarly to all works analyzed in [44], [45], which are full of plagiarism.

One of the basic working concepts studied in the analyzed papers is the notion of centrally nilpotent loop. This notion for loops on the basis of nilpotency for groups is developed and investigated in details by R. Bruck [2], [3] with the help of upper central series.

The theory of centrally nilpotent loops essentially differs from the theory of nilpotent groups. In some of his papers, [42], [40, pag. 378], [38], [14], [15] and other, Sandu offers methods to research centrally nilpotent loops, distinct from [2], [3]. They are systematized in [14] and [15], see also [46]. The notions of commutators, of commutators of weight  $n$ , of lower central series, of upper central series etc. for groups are replaced by the notions of commutator-associator, of commutator-associators of weight  $n$ , of lower central series, of upper central series etc. for loops respectively. Particularly, (ărlńń) it is given the structural construction of members of lower central series with the help of commutator-associators of type  $(\alpha, \beta)$  [14], [15], [46].

From the analyzed papers it follows that V. Ursu assumes these above-listed notions and results to him, but failing to understanding their meaning. As a result, when applying them for his researches, he makes gross mistakes and comes up with incorrect statements. Let's look at an example. For loops (in particular, for  $A$ -loops) and Moufang loops the lower central series are characterized in different ways which are equivalent with Bruck's notions. V. Ursu has not understood even this equivalence.

The analyzed works investigate the commutative Moufang loops (which by Bruck-Slaby's Theorem [3] are locally central nilpotent), the basis (minimal, independent, non-independent) of identities, quasiidentities of various varieties, quasivarieties of commutative Moufang loops, of centrally nilpotent (of class 2) of Moufang loops or  $A$ -loops. They also investigate the lattices of such varieties, quasivarieties. The basic method of researches for almost all analyzed works is the following. The above-mentioned questions, solved for groups, are transferred mechanically on the above-mentioned classes of loops. These transfers are made with gross mistakes. The main reason: the authors do not understand and not take into account the elementary properties of the statements proved for groups. The Mal'cev's results are plagiarized roughly: almost the entire work [64], [63].

Many gross blunders from the analyzed papers are corrected in various items of this paper, if it is meaningful. The items 1a) – 1o), 2a) – 2o) show falsity of the main results of papers [56], [16]. Paper [46], which is a natural continuation of this paper, proves that the variety generated by a centrally nilpotent Moufang loop (or centrally nilpotent  $A$ -loop) is finitely based. The main results of papers [56], [16] are proved wrongly.

In the end, the paper contains a scheme of conformity of the items stated below to the works from the List of References.

## 2 On basis of identities of nilpotent Moufang loops or $A$ -loops

1a). Combining techniques used by Lyndon [30] in showing that the identities of a nilpotent group are finitely based with some related ideas of Higman [25] and the results of Bruck [3] on the structure of commutative Moufang loops to prove [18] the following properties of a finitely generated commutative Moufang loop: (i) its identities are finitely based, (ii) it can be finitely presented, (iii) it is residually finite, and (iv) it has a solvable word problem.

1a<sub>1</sub>). The contents of item 1a) is the begin of paper [18], i.e. in [18] are mentioned the papers [30], [25] and [3].

1a<sub>2</sub>). In work [56] it is considerate same properties (i) – (iv) of a finitely generated centrally nilpotent Moufang loop. The structure of work, the techniques and the ideas used in [56] are a complete analog of paper [18]. In [56] it is not marked. It is follows that all this the author of [56] appropriates itself.

1a<sub>3</sub>). A basic notions and properties used in [56] for researches of properties (i) – (iv) of nilpotent (centrally nilpotent) Moufang loop are the notions of associator commutator of weight  $n \geq 1$ , of lower central series, of maximum condition for subloops and others, them properties. The author of [56] appropriates them to itself. But it does not correspond the validity. In this occasion see the item 1b<sub>4</sub>).

1b). We pass to analysis the work [56]. Literally we expose.

1b<sub>1</sub>). An algebra  $(L, \cdot, {}^{-1})$  of type  $\langle 2, 1 \rangle$  is called a Moufang loop if

$$xy \cdot zx = x(yz) \cdot x \quad (1)$$

$$y^{-1} \cdot yx = x = xy \cdot y^{-1}. \quad (2)$$

In [3] a Moufang loop is defined as a loop with one binary operation  $\langle \cdot \rangle$  satisfying the identity (1), and in such a loop any element has an inverse  $x^{-1}$  such that (2) holds as well. We show that the groupoid  $(L, \cdot)$  of the algebra  $(L, \cdot, {}^{-1})$  with identities (1) and (2) is a Moufang loop in the sense of the last definition.

All we need now is to show that each of the equations  $ax = b$ ,  $ya = b$  has a unique solution in  $(L, \cdot)$ . It is evident from (2) that these solutions are  $x = a^{-1}b$  and  $y = b^{-1}a$ .

1b<sub>2</sub>). In item 1b<sub>1</sub>) the reference on [3] not corresponds to the validity, see pages 58, 110, 115. The groupoid  $(L, \cdot)$  is not a loop. From identity (2) it follows that  $x = a^{-1}b$  and  $y = b^{-1}a$  are a solutions of equations  $ax = b$ ,  $ya = b$ . that they were an unique solutions of identities (1) and (2) it is necessary to the identities  $y \cdot y^{-1}x = xy^{-1} \cdot y = x$  or  $(x^{-1})^{-1} = x$ , or the condition that  $x \rightarrow x^{-1}$  is an one-to-one mapping "on".

1b<sub>3</sub>). Let  $L$  be a Moufang loop. The associator-commutator of weight  $n$  of elements of a nonvoid set  $M \subseteq L$  is defined by induction. Any element of  $M$  is an associator-commutator of weight 1; if  $a$  is an associator-commutator of weight  $n$  and  $b, c \in M$ , then the associator  $[a, b, c] = (a \cdot bc)^{-1} \cdot (ab \cdot c)$  and the commutator  $[a, b] = (ba)^{-1} \cdot ab$  are an associator-commutator of weight  $n + 1$ .

Let us recall from [3] the following definitions. A subloop  $H$  of the Moufang loop  $L$  is called *normal* in  $L$  if  $aH = Ha$ ,  $a \cdot bH = ab \cdot H$  for any  $a, b$  from  $L$ . The series of subloops  $L = L_1 \supset L_2 \supset \dots$ , where  $L_i$  is a normal subloop

generated by the associator-commutator of weight  $i$  is called the *lower central series* of the Moufang loop  $L$ . The subloop  $L_2$  (generated by associators and commutators from  $L$ ) is called the *associator-commutator* subloop of the Moufang loop  $L$ , and sometimes is denoted by  $A(L)$ . By the center of the Moufang loop  $L$  we mean the set

$$Z(L) = \{a \in L \mid [a, b, c] = [a, b] = 1 \text{ for all } b, c \in L\}.$$

The normal subloops are defined inductively by

$$Z_1 = Z(L), \quad Z_i(L)/Z_{i-1}(L) = Z(L/Z_{i-1}(L)) \text{ for } i \geq 1.$$

A Moufang loop  $L$  is called *nilpotent of class  $n$*  if  $L_{n+1} = \{1\}$ ,  $L_n \neq \{1\}$  or, equivalently,  $Z_n(L) = L$ ,  $Z_{n-1} \neq L$ .

1b<sub>4</sub>). From item 1b<sub>3</sub>) it follows that the notions of associator-commutator of weight  $i$ , of lower central series of the Moufang loop  $L$  are defined by author of [56], i.e. appropriates to itself. But it does not correspond the validity. Reference on [3] in item 1b) is not correct. The notions of associator-commutator of weight  $i$  and lower central series are defined in many Sandu's papers, see, for example, [42], [40, pag. 378], [38]. Confirms said and designation  $A(L)$ . Moreover, a centrally nilpotent loop is defined in [3] as  $Z_n(L) = L$ ,  $Z_{n-1} \neq L$ . The equivalence of this definition with  $L_{n+1} = \{1\}$ ,  $L_n \neq \{1\}$  is proved, for example, in mentioned Sandu's papers.

In a Moufang loop the following relations hold:

$$xL_{z,y} = x[x, y, z]^{-1}, \quad (3)$$

$$xyL_{z,t}[z^{-1}, t^{-1}] = xL_{z,t} \cdot yL_{z,t}[z^{-1}, t^{-1}], \quad (4)$$

$$xtT_z z^{-3} = xT_z \cdot yT_z z^{-3}, \quad (5)$$

$$y \cdot xL_{z,y} = yx \cdot [y, x, z]. \quad (6)$$

To prove our main result we need the following results.

1c). LEMMA 2.1 *Let  $L$  be a Moufang loop,  $k$  a positive integer such that  $a, b \in L_k$  and  $c, d, e \in L$ . Then*

$$[a, c, d] \equiv [c, d, a] \equiv [d, a, c] \pmod{L_{k+2}}, \quad (7)$$

$$[a, c, d] \equiv [c, a, d]^{-1} \equiv [a, d, c]^{-1} \pmod{L_{k+2}}, \quad (8)$$

$$[a, c, d \cdot e] \equiv [a, c, d][a, c, e] \pmod{L_{k+2}}, \quad (9)$$

$$[ab, c, d] \equiv [a, c, d][b, c, d] \pmod{L_{k+2}}, \quad (10)$$

$$[a^p, c^q, d^r] \equiv [a, c, d]^{pqr} \pmod{L_{k+2}}, \quad (11)$$

$$[ab, c] \equiv [a, c][b, c][a, b, c]^3 \pmod{L_{k+2}}, \quad (12)$$

$$[a, cd] \equiv [a, c][a, d][a, c, d]^{-3} \pmod{L_{k+2}}, \quad (13)$$

$$[a, [c, d]] \equiv [a, c, d]^{-6} \pmod{L_{k+2}}, \quad (14)$$

where  $p, q, r$  are integer numbers.

1c<sub>1</sub>). From definition of normal subloop  $L_i$  it follows that  $[a, c, d] \in L_{k+1}$  and  $[c, d, a], [d, a, c], [c, a, d] \in L_2$  for  $a \in L_k, c, d \in L = L_1$ . From identity  $[x, y, z] = [xy, z, y^{-1}]$  [3, Lemma 5.4] it follows that  $[a, c, d]^{-1} \in L_2$ . From here it follows that the relations (7), (8) are false.

1d). *Proof.* According to (3) and (6)  $c \cdot a[a, c, d]^{-1} = ca \cdot [c, a, d]$ , hence  $[a, c, d]^{-1} \equiv [c, a, d] \pmod{L_{k+2}}$ , and so  $[c, a, d] \in L_{k+2}$ . By the definition of associator,  $ca \cdot d = (c \cdot ad)[c, a, d]$ , and therefore  $d^{-1}a^{-1} \cdot c^{-1} = [c, a, d](d^{-1} \cdot a^{-1}c^{-1})$ .

Since  $[c, a, d] \in L_{k+1}$ , we have  $(d^{-1} \cdot a^{-1}c^{-1})[d^{-1}, a^{-1}, c^{-1}] \equiv (d^{-1} \cdot a^{-1}c^{-1})[c, a, d] \pmod{L_{k+2}}$ , hence  $[c, a, d] = [d^{-1}, a^{-1}, c^{-1}] \pmod{L_{k+2}}$ .

So, we obtain  $[a, c, d] \equiv [c, a, d]^{-1} = [d^{-1}, a^{-1}, c^{-1}]^{-1} \pmod{L_{k+2}}$ .

1d<sub>1</sub>). The exposed proof from item 1d) is false. We mentioned only the inaccuracy of assertion  $[c, a, d] \in L_{k+1}$  which is mentioned in  $\hat{a}$  item 1c<sub>1</sub>).

1e). According to (4), we have  $deL_{a,c} \cdot [a^{-1}, c^{-1}] = dL_{a,c} \cdot eL_{a,c}[a^{-1}, c^{-1}]$ , and, using (3)  $(de \cdot [de, c, a]^{-1}[a^{-1}, c^{-1}] = d[d, c, a]^{-1} \cdot (e[e, c, a]^{-1} \cdot [a^{-1}, e^{-1}]))$ . Therefore  $[de, c, a] = [d, c, a] \cdot [e, c, a] \pmod{L_{k+2}}$ . According to (7) and (8), we obtain (9).

1e<sub>1</sub>). From items 1c<sub>1</sub>), 1d<sub>1</sub>) it follows that the relation (9) not is proved.

1e<sub>2</sub>). The relation (9) essentially is used for prove (10), the relation (10) essentially is used for prove (11), the relation (11) essentially is used for prove (12). Consequently, the relations (10) – (12) not is proved also.

Similarly, using the commutator identity  $[x, y] = [y, x]^{-1}$ , which is true in any Moufang loop, we can prove (13). Finally, by (13), (8), (9) and (11) we have ... and (14) follows.

1e<sub>3</sub>). From above-stated it follows that the Lemma 2.1 is complete false and it proof is a senseless.

1f). LEMMA 2.2 *Let the Moufang loop  $L$  be generated by the set  $M$ . Then the factor-loop  $L_n/L_{n+1}$  is generated by those conjugacy classes which contain associator-commutators of weight  $n$  of elements of  $M$ .*

1f<sub>1</sub>). *Proof* ... By definition  $L_n$  is generated by the elements  $[x, y, z], [x, y]$ , where  $x \in L_{n-1}, y, z \in L$ . ...

1f<sub>2</sub>). After using several times the relations from Lemma 2.1, we see that ... the proof is complete.

f<sub>3</sub>). The assertion from item 1f<sub>1</sub>) is false. Correctly: by definition  $L_n$  is generated as normal subloop of loop  $L$  by the elements  $[x, y, z], [x, y]$ , where  $x \in L_{n-1}, y, z \in L$ . In such case  $L_n$  is generated as subloop by the elements  $\varphi[x, y, z], \psi[x, y]$ , where  $\varphi, \psi \in I(L)$  - the group of inner mappings of loop  $L$ , according to [3, pag. 63].

1g). THEOREM 2.3 *Any finitely generated nilpotent Moufang loop  $L$  satisfies the maximality condition for subloops.*

1g<sub>1</sub>). *Proof.* ... According to Lemma 2.2, every factor  $L_i/L_{i+1}$  is a finitely generated Abelian group. ...

1g<sub>2</sub>). Is absolutely not clear the statement from item g<sub>1</sub>) . The Theorem 2.3 not is proved.

1h). COROLLARY 2.4 *A finitely generated nilpotent Moufang loop is finitely representable.*

1h<sub>1</sub>). From items 1f<sub>2</sub>), 1e<sub>3</sub>), 1f<sub>3</sub>), 1g<sub>2</sub>) it follows that the proofs of Lemma 2.2, of Theorem 2.3, of Corollary 2.4 are false, are a senseless.

1i). LEMMA 3.2 *Every periodic local nilpotent Moufang loop is locally finite.*

1i<sub>1</sub>). In proof of Lemma 3.2 essential is used the Theorem 2.3. Them from item 1g<sub>2</sub>) it follows that the Lemma 3.2. not is proved.

1j). LEMMA 3.3 *Let  $L$  be a finitely generated nilpotent Moufang loop and let  $Z(L)$  be its centre. Then  $L/Z(L)$  is finite if and only if the associator-commutator subloop  $A(L)$  is finite.*

1j<sub>1</sub>). In proof of Lemma 3.2 essential are used the Theorem 2.3, the Lemma 3.2 and false identity (11) under  $r \neq 1$ . Them from items g<sub>2</sub>) and h<sub>1</sub>) it follows that the Lemma 3.2. not is proved.

1j<sub>2</sub>). The Lemma 3.3 and its proof are a mechanical carry of corresponding result [41, Lemma 3] for commutative Moufang loops. In [56] about it it is not marked.

1k). THEOREM 3.4 *A finitely generated nilpotent Moufang loop  $L$  is residually finite.*

1k<sub>1</sub>). *Proof.* We use induction on the class of nilpotency  $n$ . ... Let  $a \in A(L)$ . By hypothesis, there is a normal subloop  $N$  of finite index  $s$  in  $A(L)$  not containing  $a$ . ...

1k<sub>2</sub>). In proof of Theorem 3.4 are used essential are used the Theorem 2.3 and the Lemmas 3.2, 3.3. From items g<sub>2</sub>), h<sub>1</sub>) and 1j<sub>1</sub>) it follows that the Theorem 3.4 not is proved.

1k<sub>3</sub>). The last assertion from item 1k<sub>1</sub>) is not obvious, it is necessary to prove it. More particularly, it is necessary to show that if  $L$  is a nilpotent

of class  $n > 2$  Moufang loop then the associator-commutator subloop  $A(L)$  is nilpotent of class  $< n$  (strictly). In this occasion see [42, Lemma].

1k<sub>4</sub>). For commutative Moufang loops the Theorem 3.4 is proved in [43]. It in [56] is not marked.

1k<sub>5</sub>). In general, all analyzed results, except for false Lemma 2.2, are a mechanical and as have shown unsuccessful carry of results for commutative Moufang loops from Sandu's papers on centrally nilpotent Moufang loops. In [56] it is not marked About it, disappears. It confirms also designation  $A(L)$ . In Sandu's papers  $A(L)$  means the associator subloop of loop  $L$  and for commutative Moufang loops  $A(L)$  turns in commutator-associator subloop.

1l). The following Theorem 4.6 is the main results of paper [56] which for commutative Moufang loops is proved in [18]. At the proof it result [18, Theorem III] essentially are used the assertions:

a) The set of all nilpotent of class  $\leq n$  commutative Moufang loops forms a variety which is defined by identity  $((x_1, x_2, x_3), x_4, \dots, x_{2n+1}) = 1$ , where  $(x, y, z) = (xy \cdot z)(x \cdot yz)^{-1}$ . It identity is equivalent with condition  $L_n = \{1\}$ , where  $L_n$  means the associator subloop of lower central series;

b) a fully invariant subloop of free commutative Moufang loop  $F$  is normal in  $F$  as the inner mappings of a commutative Moufang loop are automorphisms. Then if  $W$  is a word subloop, generated by all values of some given set of words, then  $W$  is a normal subloop and  $F/W$  is free in the subvariety defined by the identities  $w = 1$ , for  $w$  in the given set of words.

1m). THEOREM 4.6 *The variety generated by a nilpotent Moufang loop is finitely based.*

1m<sub>1</sub>). The proof of Theorem 4.6 is erroneous, is a senseless. It is mechanical almost literal carry of proof of [18, Theorem III]. Almost literal is to hide incompetence and ignorance of many facts from theory of varieties of algebras. We confirm briefly it.

1m<sub>2</sub>). Are essentially used not proved Lemmas 2.1, 2.2, Theorem 4.3. A some identities of Lemma 2.1 are false.

1m<sub>3</sub>). In view of incompetence are ignored the corresponding assertions from item b) of 1l). In particular, is not given reason use the free Moufang loop on a countably infinite set of free generators.

1m<sub>4</sub>). The author compensates the incompetence of similar assertions from item b) of 1l) by using Lemmas 4.2, 4.3 instead of the analogous Lemmas 3, 4 from [18]. They are copied without references from [44], Propositions 5. 6. This resulted in mistakes.

1n). Finally, we cite literally.

1n<sub>1</sub>). By McKinsey [34], using Theorems 3.4 and 4.6 we obtain

1n<sub>2</sub>). Corollary 4.7 *Any nilpotent variety of Moufang loops has decidable quasiequational theory.*

1n<sub>3</sub>). *Remark.* Assume that a loop  $L$  is defined in a nilpotent variety  $\mathcal{M}$  of Moufang loops by an infinite (or infinite) number of generators  $x_1, x_2, \dots$  and a finite collection of relations  $u_1 = 1, \dots, u_n = 1$ , where the  $h_i$  are words of type  $\langle \cdot, {}^{-1} \rangle$  over the generators. The word problem consists in defining an algorithm for validity of the equality  $u = 1$  in  $L$ , where  $u$  is a word of type  $\langle \cdot, {}^{-1} \rangle$  over  $x_1, x_2, \dots$ . Since the validity of the equality  $u = 1$  in  $L$  is equivalent to the validity of the quasiidentity  $u_1 = 1, \dots, u_n = 1 \Rightarrow h = 1$  in all loops of the variety  $\mathcal{M}$ , by Corollary 4.7 we see that the word problem for finitely presented nilpotent Moufang loops is decidable.

1n<sub>4</sub>). From items 1n<sub>1</sub>), 1k<sub>2</sub>), 1m<sub>1</sub>) – 1m<sub>4</sub>) it follows that the validity of Corollary 4.7 is not proved. The McKinsey's result, published in 1943, is used in a strange manner. The definition of decidable quasiequational theory is missing.

1n<sub>5</sub>). From item 1n<sub>4</sub>) it follows that item 1n<sub>3</sub>) does not give any answer regarding the word issue for finitely presented nilpotent Moufang loops.

1n<sub>6</sub>). Actually items 1n<sub>1</sub>) – 1n<sub>3</sub>) were plagiarized. In a bit confusing form, but almost literally the author V. I. Ursu assumed to himself the Mal'cev's result from [32, Section 7], where it is shown that the word problem has a positive solution for finitely presented algebras.

1o). Taking into account the above-stated we conclude that the proofs of all statements and all results from paper [56] are often erroneous and false. Moreover, the author Ursu V. I. assumes to himself the basic working notions and the results of other authors, see, for example, items 1b<sub>4</sub>), 1j<sub>2</sub>), 1k<sub>4</sub>), 1n<sub>6</sub>). The work [56] is anti-scientific. Questions arise: what circumstances contributed to the publication of such a work and what has the author hoped for when publishing such a work?

2a). Paper [16] deserves the same appraisal as item 1o) from [56], or even worse.

2a<sub>1</sub>). [16] investigates the same questions as [18] and by item 1a<sub>2</sub>), it investigates the same questions as [56]. With respect to the techniques and methods used to investigate these questions, paper [16] is a literal copy of [56] and by item 1a<sub>2</sub>), it is a copy of [18]. [16] does not refer to these literal similarities.

2b). To hide the similarities specified in item 2a<sub>1</sub>) the authors of [16] used the following.

2b<sub>1</sub>). Replaced the standard terminology "set of identities" by an equivalent notion of "equational theory".



2b<sub>2</sub>). Similarly to items 1a<sub>3</sub>), 1b<sub>4</sub>), the authors of [16] assumed to themselves others' notions and results. For that see item 2e<sub>1</sub>).

2c). We cite the quotes of items 2c<sub>1</sub>), 2c<sub>2</sub>), 2d), 2d<sub>1</sub>), 2e).

2c<sub>1</sub>). For all the necessary notions and results from loop theory see [1], [3].

2c<sub>2</sub>). A *loop* is called an algebra  $L$  with multiplication operation and two division operations  $/, \backslash$ , where such an element  $e \in L$  exists that for all  $x, y \in L$  the following equalities hold

$$ex = xe = x; \quad xy/y = y \backslash (yx) = (x/y)y = y(y \backslash x) = x.$$

Further element  $e$  will denote the unit of loop, and  $x^{-1} = e/x$ .

2c<sub>3</sub>). The definition of Moufang loop from item 1b<sub>1</sub>) is erroneous by item 1b<sub>2</sub>). Similarly, the definition of loop from item 2c<sub>2</sub>) is erroneous. This definition is not algebraic. With such a definition the totality of all loops does not form a variety.

2c<sub>4</sub>). By [4] every  $A$ -loop is power-associative. Then  $x^{-1}x = xx^{-1} = e$ .

2d). Let  $L$  be a loop. The element  $(x, y, z) = x \backslash ((xy \cdot z)/(yz))$  is called a *left associator*, and the element  $[x, y, z] = ((xy) \backslash (x \cdot yz))/z$  is called a *right associator* of elements  $x, y, z \in L$ . The element  $[x, y] = x/(y/xy)$  is called commutator of elements  $x, y \in L$ .

2d<sub>1</sub>). For every subset  $X$  of loop  $L$  by  $[X, L]$  we denote the subloop in  $L$  generated by set

$$\{[x, y], [z, y, x], (x, y, z) | x \in X, y, z \in L\}.$$

Let  $H$  be a subloop of loop  $L$ . The set  $[H, L]$  will called a *commutator* or *commutator subloop* of subloop  $H$  in loop  $L$ . Particular,  $[L, L]$  is called *commutator* or *associant-commutant* or *derivable* of loop  $L$ . Sometimes the commutator  $[L, L]$  will be denoted as  $L'$ .

2e). Let  $L^{(0)} = L$  and  $L^{(i+1)} = [L^{(i)}, L]$ ,  $i < \omega$ . Then the descending series

$$L = L^{(0)} \supseteq \dots L^{(n)} \supseteq \dots$$

is the upper central series of an  $A$ -loop  $L$ .  $L$  is *centrally nilpotent* or *nilpotent* if  $L^{(n)} = \{e\}$  for some  $n < \omega$ .

2e<sub>1</sub>). This item is similar to items 1a<sub>2</sub>) and 1b<sub>4</sub>). The basic working notions and properties used in [16] to research the nilpotent  $A$ -loops are the notions of left associator, of right associator, of commutator of elements of an  $A$ -loop  $L$  defines in item 2d) and the definition of subset  $[X, L]$ , the equivalent property of nilpotent loop marked in items 2d<sub>1</sub>), 2e).

The authors of [16] assume to themselves the definitions and properties listed in 2d), 2d<sub>1</sub>), 2e). But this is not true. Similarly to item 1b<sub>4</sub>) the above listed notions and properties are introduced and proved in Sandu's works, for example, [42], [40, pag. 378], [38]. To hide this deceit the authors of [16] changed the terminology of these notions: the notion of associator of type  $\alpha$  was replaced by notion of right associator; the notion of associator of type  $\beta$  was replaced by notion of left associator; the notion of commutator-associator subloop was replaced by notion of commutator and others.

2f). The Lemmas 2.1 – 2.3 and Corollary 2.4 are used essentially to prove the main result of [16]. Similarly to items 1e<sub>1</sub>) – 1e<sub>3</sub>) the proofs of the listed statements are erroneous, are completely senseless.

2f<sub>1</sub>). To analyze the assertions from item 2f) the following result from [28] will be used.

2g). **Corollary 1.** *For an A-loop  $L$ , the following are equivalent:*

1.  $L$  has the inverse property, i.e.  $x^{-1}(xy) = y$  and  $(xy)y^{-1} = x$  for all  $x, y \in L$ ;

2.  $L$  has the alternative property, i.e.  $x(xy) = x^2y$  and  $(xy)y = xy^2$  for all  $x, y \in L$ ;

3.  $L$  is diassociative;

4.  $L$  is a Moufang loop.

2g<sub>1</sub>). **LEMMA 2.1.** *Let  $L$  be an A-loop,  $n$  a positive integer such that  $x, y \in L$  and  $a \in L^{(n)}$ . Then:*

$$[a, x, y] \equiv (a, x, y)^{-1} \pmod{L^{(n+2)}}, \quad (2.1)$$

$$[x, a, y] \equiv (x, a, y)^{-1} \pmod{L^{(n+2)}}, \quad (2.2)$$

$$[x, y, a] \equiv (x, y, a)^{-1} \pmod{L^{(n+2)}}, \quad (2.3)$$

2g<sub>2</sub>). **PROOF.** The elements

$$(a, x, y), (x, a, y), (x, y, a), [a, x, y], [x, a, y], [x, y, a]$$

belong to centre  $Z(L/L^{(n+2)})$  of quotient loop  $L/L^{(n+2)}$ , ...

2g<sub>3</sub>). From the definition of set  $[X, L]$  from item 2d<sub>1</sub>) and definition of lower central series from item 2e) it follows that  $(x, a, y), (x, y, a), [a, x, y], [x, a, y] \in L^{(1)}$ . Moreover, from item 2e) it follows that the listed associators are not connected directly with the definition of subloop  $L^{(n+2)}$ . Hence for  $n > 1$  the Lemmas 2.1 – 2.3 and the Corollary 2.4 are not proved, are false.

2h). **LEMMA 2.2.** *For any A-loop  $L$  and any elements  $x, y \in L$ ,  $a, b \in L^{(n)}$  the following relations hold: ... PROOF ...*

2h<sub>1</sub>). ... and similar,  $xy \cdot [xy, a] = (xy)T_a = xT_a \cdot yT_a = x[x, a] \cdot y[y, a] \equiv (xy \cdot [x, a])[y, a] \pmod{L(n+2)}$ .

2h<sub>2</sub>). ... as  $a, z] = [z, a]^{-1}$  for all  $z \in L$ .

2h<sub>3</sub>). ... hence  $[a^{-1}, x] \equiv [a, x]^{-1} \pmod{L(n+2)}$ . Used the similarly reasonings we get the relation  $a, x^{-1}] = [a, x]^{-1}$ .

2h<sub>4</sub>). The relation from item 2h<sub>1</sub>) was proved by using that  $[a, x], [b, x] \in Z(L/L^{(n+2)})$ . But by analogy with item 2g<sub>3</sub>) it is not always correct.

2h<sub>5</sub>). The relations from items 2h<sub>2</sub>), 2h<sub>3</sub>) are connected with equivalent statements from item 2g). If the loop  $L$  is not Moufang, then the relations from items 2h<sub>2</sub>) and 2h<sub>3</sub>) is not proved.

2h<sub>6</sub>). Consequently, the relations of Lemma 2.2 are not proved, some of them are false.

2i). **LEMMA 2.3.** *For any A-loop  $L$  and any elements  $x, y \in L, a, b \in L^{(n)}$  the following relations hold: ... PROOF ...*

2i<sub>1</sub>). According to relation  $[a, x] = [x, a]^{-1} \pmod{L(n+2)}$  we have ... from here

$$(x, a, y)[a, x, y][x, y, a] \equiv e \pmod{L(n+2)}.$$

By (2.1) and (2.3) we get

$$(x, a, y) \equiv [a, x, y]^{-1}[x, y, a]^{-1} \equiv (a, x, y)(x, y, a) \pmod{L(n+2)}.$$

2i<sub>2</sub>). Similar is proved and relations

$$(xy, a, z) \equiv (x, a, z)(y, a, z) \pmod{L(n+2)}, \quad (2.22)$$

$$(xy, z, a) \equiv (x, z, a)(y, z, a) \pmod{L(n+2)}. \quad (2.23)$$

2i<sub>3</sub>). To pass from the first equivalences to the second in item 2i<sub>1</sub>) it is necessary to use the property that  $L$  is a Moufang loop from item 2e<sub>1</sub>). But it is not always possible.

2i<sub>4</sub>). The relations from item 2i<sub>2</sub>) are connected with remarks from item 2g<sub>3</sub>).

2i<sub>5</sub>). According to items 2i<sub>3</sub>) and 2i<sub>4</sub>) we conclude that the relations of Lemma 2.3 are not proved, many from them are false.

2j). From Lemmas 2.1, 2.3 follows the Corollary 2.4.

2j<sub>1</sub>). From items 2g<sub>3</sub>), 2i<sub>5</sub>) it follows that Corollary 2.4 is not proved, it is false.

2k). From items 2g<sub>3</sub>), 2i<sub>5</sub>) and 2j<sub>1</sub>) it follows that the relations of Lemmas 2.1 – 2.3 and Corollary 2.4 are not proved, many from them are false.

Moreover, their proofs are not mathematical, but rather a complete non-sense.

2l). Others assertions from paper [16]: the Lemmas 2.5, 3.1 – 3.3, 4.1 – 4.4, the Theorems 2.6, 3.4, 4.6, the Corollaries 2.7, 4.5, 5.1 – 5.5 literally coincide with the corresponding statements from [56]. Thus there are only such distinctions.

2l<sub>1</sub>). Modification from item 2b<sub>1</sub>) is used.

2l<sub>2</sub>). For  $A$ -loop left or right associators, defined in item 2d), are used instead of the associator for Moufang loop defined in item 1b<sub>3</sub>).

2l<sub>3</sub>). For Lemma 3.1 the  $n$ -th right power  $x^n$  of element  $x \in L$ :  $x^n = ((\dots((x \cdot x) \cdot x) \dots) \cdot x)x$  ( $n$  steps) is defined. By [4] every  $A$ -loop  $L$  is power-associative, i.e. every element in  $L$  generates an associative subloop. Hence the defined notion of right power is without sense.

2m). Clearly, all of the above-mentioned mistakes and all non-senses for Moufang loops in [56] hold for  $A$ -loops considered in [16].

2n). Items 2k) and 2m) confirm the estimation and the characteristic from item 2a) for [16].

2o). In papers [14], [15] is specified on wrongly and anti-scientific character of works [16], [56].

### 3 On V. Ursu and A. Covalschi compositions

3a). The main result of work [62] is the following statement.

3a<sub>1</sub>). **Theorem 1.** *A quasivariety of commutative Moufang loops is solvable, i.e. to contain only solvable loops, if and only if each of its loops is different from its associator.*

3b). Before analyzing the proof of Theorem 1 we cite the quotes 3b<sub>1</sub>), 3c), 3d), 3e) – 3e<sub>3</sub>).

3b<sub>1</sub>). Now we will make an overview of some notions connected with Moufang loops, some of which may be found in [3] or [1].

3c). An algebra  $(L, \cdot, {}^{-1})$  of  $\langle 2, 1 \rangle$  type, whose elements and main operations satisfy the identities  $x^{-1} \cdot xy = y = yx \cdot x^{-1}$ ,  $xy = yx$ ,  $xy \cdot zx = x(yz \cdot x)$  is called a *commutative Moufang loop*.

3c<sub>1</sub>). The definition of commutative Moufang loop from item 3c) does not correspond to the definition from [3], [1].

3c<sub>2</sub>). The definition of commutative Moufang loop from item 3c) is not correct, is not algebraical. For more details see items 1b<sub>1</sub>), 1b<sub>2</sub>).

3d). *The quasivariety of commutative Moufang loops is closed relative to the limit of the direct spectrum.*

3d<sub>1</sub>). The statement from item 3d) is not proved. According to [32] to prove it is necessary to consider the quasiidentities, but not the atomic formulas and universal formulas.

3e). A collection  $\Lambda = \langle I, L_i, \varphi_{ij} \rangle$ , consisting of an oriented set  $\langle I, \leq \rangle$ , a family  $\{L_i | i \in I\}$  of commutative Moufang loops  $L_i$  and a family  $\{\varphi_{ij} | i, j \in I, i \leq j\}$  of homomorphisms  $\varphi_{ij} : L_i \rightarrow L_j$  is called a direct spectrum if  $\varphi_{ij}$  is an identical application and  $\varphi_{ik} = \varphi_{jk} \varphi_{ij}$  for every  $i, j, k \in I, i \leq j \leq k$ .

3e<sub>1</sub>). We denote by  $L_\infty$  the quotient set of the set  $\cup_{i \in I} L_i x \{i\}$  by the equivalence relation  $\equiv$ :

$$(a, i) \equiv (b, j) \Leftrightarrow \exists k \in I (i \leq k \vee j \leq k \vee \varphi_{ik}(a) = \varphi_{jk}(b)).$$

3e<sub>2</sub>). Let  $[a, i]$  be the coset with respect to the equivalence relation  $\equiv$  that contains the element  $(a, i) \in L_i x \{i\}$ . On the set  $L_\infty$  we define operations of commutative Moufang loops in the following way:

$$[a, i]^{-1} = [a^{-1}, i], \quad [a, i] \cdot [b, j] = [\varphi_{ik}(a) \cdot \varphi_{jk}(b), k], \text{ where } i \leq k, j \leq k.$$

3e<sub>3</sub>). We can check without any difficulty the correctness of these operations and whether they satisfy the relations of defining the commutative Moufang loop. The obtained commutative Moufang loop will be called *the direct limit over the direct spectrum*  $\Lambda$  and will be denoted by  $\lim \Lambda$  [24].

3e<sub>4</sub>). The definition of direct spectrum from item 33) does not correspond to the classical definition of direct spectrum. The authors of [62] have not understood or purposely changed the correct expression "a family  $\{\varphi_{ij} | i, j \in I, i \leq j\}$  of isomorphisms  $\varphi_{ij} : L_i \rightarrow L_j$ " for "a family  $\{\varphi_{ij} | i, j \in I, i \leq j\}$  of homomorphisms  $\varphi_{ij} : L_i \rightarrow L_j$ ". Thus, the basic sense of direct spectrum is lost, leading to gross mistakes. Further, instead of " $\varphi_{ij}$  is an identical application", " $\varphi_{ii}$  is an identical application" is necessary.

3e<sub>5</sub>). The inscription  $L_i x \{i\}$  from items 3e<sub>1</sub>), 3e<sub>2</sub>) is not clear.

3e<sub>6</sub>). Between items 3e) and 3e<sub>1</sub>) it is necessary to add the expression "To distinguish elements  $a \in L_i$  from elements Others loops  $L_j, i \neq j$ , we shall use the notation  $(a, i)$ ".

3e<sub>7</sub>). The statements from items 3e<sub>2</sub>), 3e<sub>3</sub>) are useless, as they follow directly from the correct definition of direct spectrum according to item 3e<sub>4</sub>).

3f). Now let us analyze the proof of Theorem 1. We cite items 3f<sub>1</sub>), 3f<sub>2</sub>).

3f<sub>1</sub>). let  $\mathcal{R}$  be a quasivariety each loop of which is different from its associator, and let us suppose that the quasivariety  $\mathcal{R}$  contains unsolvable

loops. Then it is clear that the free loop  $F$  of the quasivariety  $\mathcal{R}$  with the countable set  $\{x_1, x_2, \dots\}$  of free generators  $x_1, x_2, \dots$ , is unsolvable.

Let us analyze the direct spectrum  $\tau = \langle N, F_i, \varphi_{ij} \rangle$  consisting of the set of integers  $N = \{0, 1, 2, \dots\}$ , the loop's isomorphic copies  $F_i$  of the loop  $F_0 = F$  with the sets  $\{x_1^i, x_2^i, \dots\}$ ,  $i = 1, 2, \dots$ , of free generators and the homomorphisms  $\varphi_{ij}$  defined in the following way:

$$\varphi_{ij} = \varphi_{j-l_j} \cdots \varphi_{i+1+i+2} \varphi_{i+2},$$

where the homomorphisms  $\varphi_{k_{k+1}} : F_k \rightarrow F_{k+1}, \dots$ , are defined on generators by the equalities

$$\begin{aligned} \varphi_{0_{k+1}}(x_j) &= [x_{3j-2}^l, x_{3j-1}^l, x_{3j}^l], \\ \varphi_{k_{k+1}}(x_j^k) &= [x_{3j-2}^{k+1}, x_{3j-1}^{k+1}, x_{3j}^{k+1}], \quad j = 1, 2, \dots \end{aligned}$$

3f<sub>2</sub>). Thus, generators of the loop  $\lim \tau$  belong to the associator  $(\lim \tau)'$  and therefore, we have  $\lim \tau = (\lim \tau)'$ . In such a way the non-identity loop  $\lim \tau$  from the quasivariety  $\mathcal{R}$  coincides with its associator, but this contradicts the hypothesis. This proves the theorem.

3f<sub>3</sub>). According to item 3f<sub>1</sub>) the loop  $F_i$  is generated by set of free generators  $\{x_1^i, x_2^i, \dots\}$ . It is clear that the set  $\{\varphi_{i+1} x_1^i, \varphi_{i+1} x_2^i, \dots\}$  generates the subloop  $\varphi_{i+1} F_i \subseteq F_{i+1}$ . Then  $\{\varphi_{i+1} x_1^i, \varphi_{i+1} x_2^i, \dots\} \cup \{x_1^{i+1}, x_2^{i+1}, \dots\}$  generates a subloop of direct limit  $\lim \tau$  of direct spectrum  $\tau$  of commutative Moufang loops  $F_i$ . Let  $H$  be the subloop of  $\lim \tau$  generated by set  $\{\varphi_{i+1} x_1^i, x_1^{i+1}, x_2^{i+1}, x_3^{i+1}\}$ . By [3]  $H$  is centrally nilpotent. Then by [37]  $H' \subseteq \Phi(H)$ , where  $\Phi(H)$  denote the Frattini subloop. By item 3f<sub>1</sub>) we have  $\varphi_{i+1} x_1^i = (x_1^{i+1}, x_2^{i+1}, x_3^{i+1})$ . We get a contradiction:  $\varphi_{i+1} x_1^i$  is a generator,  $(x_1^{i+1}, x_2^{i+1}, x_3^{i+1})$  is non-generator.

3f<sub>4</sub>). From item 3f<sub>3</sub>) it follows that the equality  $\lim \tau = (\lim \tau)'$  from item 3f<sub>2</sub>) is not proved.

3f<sub>5</sub>). From item 3f<sub>4</sub>) it follows that the Theorem 1 is not proved.

3f<sub>6</sub>). Similarly to Theorem 1 it is shown that Theorem 2 of [62] is not proved.

3g). The Theorems 1, 2 are used essentially in the proofs of Corollaries 1 – 5 of [62]. Hence, the Corollaries 1 – 5 of [62] are not proved.

3h). In particular, the following statement is not proved.

**Corollary 5.** *There exist 3-periodic commutative Moufang loops which contain subloops that coincide with their associator in the given loop.*

3i). According to items 3f<sub>5</sub>), 3f<sub>6</sub>), 3g), 3h) we conclude that the proofs of all assertion from paper [62] are erroneous, are completely senseless. The work [62] is written carelessly. For example, Corollaries 2 and 4 literally

coincide. We appreciate that paper [62] similarly to papers [56], [16] from items 1o), 2a).

4a). To analyze paper [13] we cite the items 4a<sub>1</sub>), 4b) – 4b<sub>4</sub>).

4a<sub>1</sub>). **Theorem.** *If the loop  $L$  contains an infinite cyclic group  $Z$  and does not contain an infinity of  $p_i$ -periodic elements ( $p_i$  denote a prime number), then the quasiidentity  $qL$  generated by the loop  $L$  has an infinite and independent basis of quasiidentities.*

4a<sub>2</sub>). **Theorem 1** [5]. *If the group  $L$  contains an infinite cyclic group  $Z$  and does not contain an infinity of  $p_i$ -periodic elements ( $p_i$  denote a prime number), then the quasiidentity  $qL$  generated by the group  $L$  has an infinite and independent basis of quasiidentities.*

4a<sub>3</sub>). From items 4a<sub>1</sub>), 4a<sub>2</sub>) it follows that the main result of [13], the Theorem, literally coincides with Theorem 1 from [5]. The only difference is that the word "group" was replaced by word "loop".

4a<sub>4</sub>). Lemma 1 and its proof from [13] is identic with the Lemma 1 and its proof from [5].

4a<sub>5</sub>). Lemmas 2 and 3 and their proofs from [13] literally coincide with Lemma 3 and 2, respectively, and their proof from [5]. The only difference is that identity  $u(x_1, x_2, \dots, x_n) = e$  for loops in signature  $(\cdot, /, \backslash, e)$  was replaced by  $u_1(y_1, y_2, \dots, y_k) = u_2(z_1, z_2, \dots, z_m)$  for quasigroups in signature  $(\cdot, /, \backslash)$ .

4a<sub>6</sub>). In proof of Lemma 3 in [5] is applied the Dik's Theorem, see item 7c<sub>3</sub>). Thus it is essential used the statement from item 7a<sub>1</sub>). In proof of Lemma 2 in [13] is used the Dik's Theorem for  $A$ -loops similarly to this. But it is a roughest mistake as for  $A$ -loops not is known the analogue of item 7a<sub>1</sub>)..

4a<sub>7</sub>). The proof of Theorem 1 from item 4a<sub>2</sub>) essential use the Lemmas 1 – 3 from [5]. Respectively, the proof of Theorem from item 4a<sub>1</sub>) essential use the Lemmas 1 – 3 from [13]. Moreover, the proof of Theorem from item 4a<sub>1</sub>) literally coincides with the proof of Theorem 1 from item 4a<sub>2</sub>). The only differences are mentioned in items 4a<sub>3</sub>) and 4a<sub>5</sub>).

4a<sub>8</sub>). According to items 4a<sub>3</sub>) – 4a<sub>7</sub>) we conclude that Section 2 (and Section 1) of [13] is copied literally from [5], thus the roughest mistakes are admitted, see the item 4a<sub>6</sub>). Further, we cite from [13] items 4b) – 4b<sub>4</sub>).

4b). **Corollary 2.** *Every torsion-free nilpotent loop has an infinite and independent basis of quasiidentities.*

4b<sub>1</sub>). **3 Applications 1.** From local Mal'cev Theorem's the following coverage criterion of quasivarieties results: If the quasivariety  $M$  has an independent and infinite basis of quasiidentities, then  $M$  has an infinity of

coverages. The detailed proof of this statement can be found, for instance, in [6].

4b<sub>2</sub>). According to Corollary 2 and the coverage criterion of quasivarieties, we obtain the following statement.

4b<sub>3</sub>). *If  $L$  is a torsion free nilpotent loop of any rank, then the quasivariety  $qL$  has infinity of coverages in the lattices of loop quasivarieties.*

4b<sub>4</sub>). Finally, I would like to thank the university teacher V. I. Ursu for his input to the final editing, as well as for the precious remarks in the construction of loops with infinite independent bases of quasiidentities.

4b<sub>5</sub>). Item 4b<sub>1</sub>) is literally copied from [5].

4b<sub>6</sub>). The statements from items 4b) and 4b<sub>3</sub>) belong to the author of [13]. But in such formulations they are false, incorrect. It is necessary to add the conditions that the nilpotent loop contains an infinite cyclic group and the condition with respect to  $p$ -subloops.

4b<sub>7</sub>). The items 4b), 4b<sub>3</sub>) and 4b<sub>4</sub>) are a reason to consider an example of nilpotent loop. But this example is also copied, without mentioning it: however it is the example II.4.20 from [12] of centrally nilpotent commutative Moufang loop of class 2.

4c). According to items 4a<sub>1</sub>) – 4b<sub>7</sub>) we conclude that without false assertions of items 4b) and 4b<sub>3</sub>) all results of work [13] are a literal and mechanical transfer of group results from [5] on quasigroups and loops. Some insignificant differences are specified in items 4a<sub>3</sub>), 4a<sub>5</sub>). Paper [5] is not mentioned.

4c<sub>1</sub>). Paper [13] investigates quasivarieties of loops. It is not clear why the reviewers of paper [13] and the university teacher V. I. Ursu, mentioned in item 4b<sub>4</sub>), have not asked the question: what is the situation with similar results in the group theory?

5a). To analyze paper [61] we cite items 5a<sub>1</sub>), 5a<sub>3</sub>), 5b).

5a<sub>1</sub>). Let  $L_0 \supseteq L_1 \supseteq \dots \supseteq L_n \supseteq \dots$  be the central descendant range of the loop  $L$ , i.e.  $L_0 = L$ , and for any  $n > 0$ ,  $L_n/L_{n+1}$  is the subloop from the centre of the factor loop  $L/L_{n+1}$ . The loop  $L$  is called *nilpotent* (or *centrally nilpotent*) if there is such a natural number  $n$  that  $L_n = \{e\}$ . The least natural number  $n$  for which  $L_n = \{e\}$  is called the *class of nilpotence* of  $L$ .

5a<sub>2</sub>). The analyzed paper [61] uses essentially the results for nilpotent  $A$ -loop from [16], see items 2c<sub>2</sub>), 2d), 2d<sub>1</sub>), 2e), 2e<sub>1</sub>). The definition of nilpotent loop from item 5a<sub>1</sub>) differs from the corresponding definition from [16], see item 2e). The equivalence of these definitions not is mentioned. In reality, these definitions are copied from Sandu's papers and this is covered:



for details see items  $1b_4$ ,  $2c_1$ ).

$5a_3$ ). Let  $L$  be a nilpotent  $A$ -loop of class 2 and  $x, y$  and  $z$  elements from  $L$ . The element  $(x, y, z)x \setminus ((xy \cdot z)/(yz))$  is called the associator of the elements  $x, y, z \in L$ ; the element  $[x, y] = x/(y/xy)$  is called the commutator of elements  $x, y \in L$ .

$5a_4$ ). The subloop of loop  $L$ , generated by the set

$$\{[x, y], (x, y, z) \mid x, y, z \in L\}$$

is called the *commutator* of the  $A$ -loop  $L$  and will be denoted by  $L'$ .

$5a_5$ ). The definition of commutator from item  $5a_4$ ) differs From the corresponding definition from [16].

$5b$ ). According to [16] in any nilpotent  $A$ -loop of class 2 the following identities hold true

$$\begin{aligned} [x \cdot y, z] &= [x, z] \cdot [y, z], \\ [x, y \cdot z] &= [x, y] \cdot [x, z], \\ (x, y, z) &= (y, x, z) \cdot (x, z, y), \\ (x, y, x) &= e, \\ (x \cdot y, z, t) &= (x, z, t) \cdot (y, z, t), \\ (x, y \cdot z, t) &= (x, y, t) \cdot (x, z, t), \\ (x, y, z \cdot t) &= (x, y, z) \cdot (x, y, t) \end{aligned}$$

which will be used further by default.

$5b_1$ ). The identities from item  $5b$ ) in [16] is not proved by items  $2h_6$ ),  $2i_5$ ). For this, see item  $5a_2$ ).

$5b_2$ ). Identity  $(x, y, x) = e$  from item  $5b$ ) in [16] is not considered.

$5b_3$ ). Using the equivalent assertions from item  $2g$ ) and the identities from  $5b$ ) it is possible to show that the identity  $(x, y, x) = e$  from item  $5b$ ) is not correct in any  $A$ -loop.

$5c$ ). To analyze the main result of [61] we cite items  $5c_1$ ) –  $5c_5$ ).

$5c_1$ ). **Theorem.** *The quasiidentities of the finitely generated nilpotent  $A$ -loop have a finite basis of quasiidentities if and only if it is a finite Abelian group.*

**Proof.** ...

$5c_2$ ). The proof of sufficiency of Theorem is unsuccessful. The sufficiency directly follows from Ol'shanski result on finitely based of quasiidentities of finite group.

$5c_3$ ). Let  $L$  be a nilpotent  $A$ -loop. ...

According to [16], the periodical nilpotent  $A$ -loop is locally finite.

5c<sub>4</sub>). According to Theorem 2.6 [16], the loop  $L$  satisfies the condition of maximality for subloops ...

5c<sub>5</sub>). Hence, the nilpotent  $A$ -loop  $L$  is finite. Suppose that  $L$  is non-associative or non-commutative. Then  $L$  contains a non-associative or non-commutative  $p$ -subloop  $H$ .

5c<sub>6</sub>). The proofs of assertions from items 5c<sub>4</sub>), 5c<sub>5</sub>) in [16] are erroneous by item 2l).

5c<sub>7</sub>). For  $A$ -loops it is necessary precisely to determine the concept of a  $p$ -loop as unlike groups for  $A$ -loops two non-equivalent concepts are available [26]. For validity of assertion from 5c<sub>5</sub>) it is necessary to show that the totality of all  $p$ -elements of  $L$  form a subloop.

5d). The proof of Theorem contains the Lemmas 1 – 4, which investigate the free loop  $F_n$  of the rank  $m$  with free generators  $x_1, x_2, \dots, x_n$  in the quasivariety, generated by loop  $N$ . By definition  $N = lp(a_1, a_2)$ , where  $a_1 = a, a_2 = ab^{p^{\beta-1}}, a, b \in L$ .

5d<sub>1</sub>). The basis of proofs of Lemmas 1 – 4 is the transformation of associators of elements of free loop  $F_n$  with the help of identities from item 5b). The listed identities hold for nilpotent  $A$ -loop of class 2. To use these identities for  $F_n$  is a gross mistake.

5d<sub>2</sub>). To correct the specified mistakes in item 5d<sub>1</sub>) it is necessary to show that loop  $F_n$  is nilpotent of class 2. It is not clear how it is possible to make such a variant of definition of nilpotent loop from item 5a<sub>1</sub>).

5e). Lemma 1 is the basis to prove Theorem. But its proof contains a gross mistake.

5e<sub>1</sub>). Further we will mark as  $F_n$  (or  $F_n(x_1, \dots, x_n)$ ) the free loop in the quasivariety  $Q(N)$ , of the rank  $m$  (with free generators  $x_1, \dots, x_n$ ).

**Lemma 1.** *The commutator  $F'_n$  of the  $Q(N)$ -free loop  $F_n(x_1, \dots, x_n)$  is a free Abelian group with the exponent  $p$ , with the following free generators:*

a)  $(x_i, x_i, x_j), (x_i, x_j, x_j), 1 \leq i < j \leq n; (x_i, x_j, x_k), 1 \leq i < j < k \leq n; [x_i, x_j], 1 \leq i < j \leq n,$

**if  $F_n$  is non-commutative;**

b)  $(x_i, x_i, x_j), (x_i, x_j, x_j), 1 \leq i < j \leq n; (x_i, x_j, x_k), 1 \leq i < j < k \leq n;$

**if  $F_n$  is commutative.**

5e<sub>2</sub>). *Proof.* To prove the lemma it is sufficient to show that any relation of equality between the generators of the group  $F'_n$  shown in a) (for b) the procedure is similar) is a trivial identity in the variety  $V(N)$ .

5e<sub>3</sub>). Indeed, let

$$\prod_{1 \leq i < j \leq n} (x_i, x_j, x_k)^{\alpha_{ij}} \cdot \prod_{1 \leq i < j \leq n} (x_i, x_i, x_j)^{\beta_{ij}}.$$

$$\prod_{1 \leq i < j < k \leq n} (x_i, x_j, x_k)^{\gamma_{ijk}} \cdot \prod_{1 \leq i < j \leq n} (x_i, x_j)^{\delta_{ij}} = e \quad (1)$$

be a such a relation of equality.

5e<sub>4</sub>). The assertion from item 5e<sub>2</sub>) is correct theoretically. But it applies only for elements of free  $A$ -loop  $F_n$  of variety  $V(N)$  written down in canonical, normal form with respect to free generators of  $F_n$ . This problem is in detail described in [12] for various varieties of loops.

According to item 5c<sub>5</sub>) the loop  $N$  is a  $p$ -loop. [26] proves that if  $p$  is an odd prime and  $Q$  is a finite commutative  $A$ -loop, then  $Q$  is centrally nilpotent. We know now that no nonassociative finite simple commutative  $A$ -loop of order less than  $2^{12}$  exists [27].

5e<sub>5</sub>). Unlike free groups, the given assertions show the supercomplexity of a problem from item 5e<sub>4</sub>) for free  $A$ -loop  $F_n$ .

5e<sub>6</sub>). From item 5e<sub>5</sub>) it follows that the equality (1) from item 5e<sub>3</sub>) there is not enough for the proof of Lemma 1. Consequently, the Lemma 1 is not proved. To analyze the proofs of Lemmas 2 - 4 it is not meaningful.

5e<sub>7</sub>). From items 5b<sub>3</sub>), 5c<sub>6</sub>), 5c<sub>7</sub>), 5d<sub>1</sub>), 5d<sub>2</sub>), 5e<sub>6</sub>) it follows that the proof of Theorem is senseless.

5f). Further, we cite items 5f<sub>1</sub>) – 5g<sub>1</sub>).

5f<sub>1</sub>). Directly from the Theorem two corollaries follow.

5f<sub>2</sub>). **Corollary 1.** *If a finite nilpotent  $A$ -loop is not commutative or associative, then it has no basis of quasiidentities from a finite number of variables.*

5f<sub>3</sub>). **Corollary 2.** *If  $L$  is a finitely generated nilpotent  $A$ -loop, then  $Q(L) = V(L)$  if and only if  $L$  is a finite Abelian group.*

5g). Similarly to [60] we will prove the following.

5g<sub>1</sub>). **Corollary 3.** *The lattice of subquasivarieties of the variety generated by a finitely generated nilpotent and non-associative or non-commutative  $A$ -loop has the power of the continuum.*

5h). From items 5e<sub>7</sub>), 5f<sub>1</sub>) it follows that the Corollaries 1 – 3 are not proved. Consequently, all assertions from paper [61] are not proved, many from them are false.

## 4 On V. Ursu compositions

6a). Paper [60], mentioned in item 5g) is characterized similarly as [61] in item 5h).

7a). Many works of Budkin A. I., Gorbunov V. A. and others investigate questions about cardinality, independence of basis of quasiidentities and

describe the lattices of subquasivarieties of quasivarieties of various classes of groups, particularly, of nilpotent groups. To find an answer to these questions, the following result is used essentially.

7a<sub>1</sub>). Every  $n$ -generated subgroup of group

$$\langle x, x_1, \dots, x_{2n}; x^{p_i}[x_1, x_{n+1}] \dots [x_n, x_{2n}] \rangle, \quad i \in \mathfrak{N},$$

is free. [6, Lemma 3], [5].

7a<sub>2</sub>). Many works of Ursu V. I. examine similar questions, mentioned in item 7a), for quasivarieties of centrally nilpotent commutative Moufang loops and for quasivarieties of centrally nilpotent Moufang loops. In general the proofs of these questions in Ursu's works have the following form. The statements on quasivarieties of groups and their proofs contain the following replacements: commutator in group  $\Rightarrow$  associator in commutative Moufang loop or commutator-associator in Moufang loop; nilpotent group  $\Rightarrow$  centrally nilpotent Moufang loop. As a result, such mechanical replacements result in none-sense. Moreover, the assertions from items 5e<sub>4</sub>) and 5e<sub>5</sub>) for  $A$ -loops hold for commutative Moufang loop, for centrally nilpotent Moufang loops at the specified transitions from groups to such loops. We present some examples, analyzing a little Ursu's works.

7b). We cite items 7b<sub>1</sub>) – 7b<sub>5</sub>) to analyze paper [53].

7b<sub>1</sub>) (pag. 106). Fix some integer  $m \geq 2$  and denote by  $\mathfrak{N}$  the variety defined in class of all commutative loops Moufang by the following identities:

$$(\forall x)(x^{3^m} = 1), (\forall x)(\forall y)(\forall z)([x, y, z]u \cdot v = [x, y, z] \cdot uv).$$

7b<sub>2</sub>). Now we denote by  $B$  that loop of  $\mathfrak{N}$  represented as:

$$B = (a, b, c \parallel a^{3^m} = b^{3^m} = c^{3^{m-1}} = 1, \quad a^{3^{m-1}} = [a, b, c]).$$

7b<sub>3</sub>). We denote by  $Q(B)$  the quasivariety generated by  $B$ . In the following via  $\mathfrak{M}$  we shall understand the class of all loops of  $Q(B)$ , which does contain the subloops isomorphic to  $B$ .

We investigate the commutative loops Moufang  $A_n$ ,  $n = 1, 2, \dots$ , which are represented in the variety  $\mathfrak{N}$  as follows:

$$\begin{aligned} A_n &= (x_1, \dots, x_n \parallel [x_i, x_j, x_k] = \\ &= \begin{cases} 1, & \text{if } 2|i, 2|j, 2|k \text{ or } 2 \nmid i, 2 \nmid j, 2 \nmid k, \\ x_k^{3^{m-1}}, & \text{otherwise.} \end{cases} \\ &1 \leq i < j < k \leq n. \end{aligned}$$

7b<sub>4</sub>). According to Dick's theorem [33] the application  $\phi : A_n \rightarrow B$  for which  $x_n^\phi = a$ ,  $x_{n-1}^\phi = b$ ,  $x_{n-2}^\phi = c$ ,  $x_i^\phi = 1$ ,  $i = 1, \dots, n-3$  is a loop morphism from  $A_n$  into  $B$ .

7b<sub>5</sub>). **Theorem 1.** *The unique maximal quasivariety  $\mathfrak{M}$  of the lattice  $L_q Q(B)$  is not generated by a finite loop.*

**Theorem 2.** *The set of all quasivarieties each of them generated by a finite loop of the variety  $\mathfrak{N}$  does not form a sublattice in the lattice of all quasivarieties of  $\mathfrak{M}$ .*

7c). The Dick's theorem [33, Theorem V.11.5] examines only an algebraic system given by defining relations. The loops  $A_n$  and  $B$  are defined in items 7b<sub>2</sub>) and 7b<sub>3</sub>) with the help diagrams (see [33, pag. 251]. Hence, the assertion from item 7b<sub>4</sub>) is false. For this see also item, 7c<sub>2</sub>) containing a variant of Dick's theorem for groups.

7c<sub>1</sub>). The assertion from item 7b<sub>4</sub>) is essentially used to prove the main results of paper [53], the Theorems 1, 2. Hence, Theorems 1, 2 are not proved.

7c<sub>3</sub>). **Dick's theorem** [7]. *Let the group  $G$  has in given quasivariety  $\mathfrak{N}$  the representation  $G = gr(\{x_i | i \in I\} \parallel \{r_j(x_{j_1}, \dots, x_{j_{i(j)}}) = 1 | j \in J\})$ . Assume that  $H \in \mathfrak{N}$  and the group  $H$  contains a set of elements  $\{g_i | i \in I\}$  such that for every  $j \in J$  the equality  $r_j(g_{j_1}, \dots, g_{j_{i(j)}}) = 1$  is true in  $H$ . Then the mapping  $x_i \rightarrow g_i (i \in I)$  proceeds up to homomorphism  $G$  in  $H$ .*

7d). Let's now examine others non-senses from [53], for example, the equality  $a^{3^{m-1}} = [a, b, c]$  from item 7b<sub>2</sub>). The item 7c<sub>1</sub>) is sufficient to conclude that all results of [53] are not proved, the work [53] is a nonsense.

8a). Similarly to group's theory and using the methods described in item 7a<sub>2</sub>) Ursu's works [54], [55], [58], [59] investigate some questions for quasivarieties of commutative Moufang loops. Let's consider only a few roughest mistakes contained in papers [54], [55], [58], [59]. They are sufficient to conclude that all results and their proofs from [54], [55], [58], [59] are erroneous, often are without sense.

8a<sub>1</sub>). We cite items 8b), 8b<sub>1</sub>), 8c), 8c<sub>1</sub>), 8c<sub>2</sub>), 8c<sub>4</sub>), 8d), 8f), 8f<sub>1</sub>), 8f<sub>2</sub>), 8g), 8g<sub>1</sub>) to analyze papers [54], [55], [58], [59].

8b). A commutative Moufang loop (see [3]) is an algebra  $L$  with the unary operation  $^{-1}$  and a binary operation  $\cdot$  in which for any  $x, y, z \in L$  we have

$$xy = yx, \quad xy \cdot zx = x(yz) \cdot x, \quad x^{-1} \cdot xy = y \quad [54], [58].$$

8b<sub>1</sub>). A groupoid  $L(\cdot)$  is called a *quasigroup* if, for any  $a, b \in L$ , each of the equations  $ax = b$ ,  $ya = b$  has a unique solution. A quasigroup with unity is called a *loop* [55].

8b<sub>2</sub>). The definition from item 8b) is incorrect by item 1b<sub>2</sub>) .

8b<sub>3</sub>). The definition from item 8b<sub>1</sub>) is not algebraic by [33].

8b<sub>4</sub>). The definitions of quasigroup, loop from item 8b<sub>1</sub>) (from [55]) does not allow to use the powerful apparatus of quasivarieties, of varieties loops, particular, of their free loops according to item 8b<sub>3</sub>).

8c). We introduce the following notation [54], [58]:

$F_n = F_n(x_1, \dots, x_n)$  is the free commutative Moufang loop of order  $n$  generated by the free elements  $x_1, \dots, x_n$ ;

$\mathfrak{N}_2$  is the class of all commutative Moufang loops with nilpotency class  $\leq 2$  defined by the identity

$$(xy \cdot z)(x \cdot yz)^{-1} \cdot uv = ((xy \cdot z)(x \cdot yz)^{-1} \cdot u)v;$$

$\mathfrak{N}_{2,3^k}$  is the variety defined in  $\mathfrak{N}_2$  by the identity  $x^{3^k} = 1$ , where  $k \neq 0$  is a natural number;

$F_n(\mathfrak{M}) = F_n(\mathfrak{M}; x_1, \dots, x_n)$  is the free commutative Moufang loop of rang  $n$  of the variety  $\mathfrak{M} \subseteq \mathfrak{N}_2$ ;

$Q(L)$  is the quasivariety generated by the commutative Moufang loop  $L$ ;

$L_q\mathfrak{M}$  is the lattice of subquasivarieties of the quasivariety  $\mathfrak{M} \subseteq \mathfrak{N}_2$ ;

$T = Q(F_3), T_k = Q(F_3(\mathfrak{N}_{2,3^k}))$ ;

$Z_{p^k}$  is the cyclic group of order  $p^k$ , where  $p$  is a prime.

8c<sub>1</sub>). Usually, the elements of the commutative Moufang loop  $F_n(x_1, \dots, x_n)$  are called the words of the variables  $x_1, \dots, x_n$  and the elements of  $F'_n$  are called associator words.

8c<sub>2</sub>). For each  $k = 1, 2, 3, \dots$  let  $\mathfrak{N}_{(2,3^k)}$  be the sub-quasivariety of  $\mathfrak{N}_{2,3^k}$  defined by the quasi-identities:

$$x^{3^{k-1}} = 1 \rightarrow [x, y, z] = 1, \quad (4)$$

$$x^3 = [x_1, x_2, x_3] \dots [x_{3n-2}, x_{3n-1}, x_{3n}] \rightarrow x^3 = 1, \quad (5)$$

where  $n$  runs over the natural numbers. Let  $\mathfrak{N}_{(2)}$  be the quasivariety of  $\mathfrak{N}_2$  defined by the quasi-identities:

$$x^9 = 1 \rightarrow x^3 = 1,$$

$$x^3 = 1 \rightarrow [x, y, z] = 1, \quad (7)$$

$$x^p = 1 \rightarrow x = 1, \quad (8)$$

where  $p$  runs over all the prime numbers not equal to 3.

8c<sub>3</sub>). **Proposition.** *A quasivariety  $\mathfrak{N}_{(2,3^k)}$ ,  $k = 1, 2, 3, \dots$ , contain a non-associative commutative Moufang loops if and only if  $k = 1$ . In such*

case the quasivariety  $\mathfrak{N}_{(2,3)}$  coincides with variety  $\mathfrak{N}_{2,3}$  of all centrally nilpotent commutative Moufang loops of class  $\leq 2$   $\mathfrak{N}_2$  which satisfy the identity  $x^3 = 1$ .

**Proof.** Let  $L$  be a centrally nilpotent Moufang loop, then  $L' \neq L$ . By [37] the associator suploop  $L'$  belongs to Frattini subloop  $\Phi(L)$ . Hence the quotient loop  $L/L'$  is non-unitary. By [3]  $a^3 \in Z(L)$  for all  $a \in L$ . Then the subloop  $L^3$  generated by set  $\{a^3 | a \in L\}$  is normal in  $L$ . Hence the quotient loop  $L/L^3L'$  is an elementary abelian 3-group. In such case  $L/L^3L'$  decompose into a direct product of cyclic groups of order 3 and the Frattini subloop  $\Phi(L/L^3L')$  as intersection of all maximal proper subloops of  $L/L^3L'$  ([3, Theorem VI.2.1]) is equal to unit  $L^3L' = \bar{1}$ . By [3, Lemma VI.2.1]  $\Phi(L)/L^3L' \subseteq \Phi(L/L^3L') = \bar{1}$ , From here it follows that  $L^3L' = \Phi(L)$ .

The free loop  $F(\mathfrak{N}_{(2,3^k)}; x_1, x_2, \dots, x_{3n}) = F_{3n}(\mathfrak{N}_{(2,3^k)})$  satisfies the identity  $x^{3^n} = 1$ . Then by [37] it is finite. Let us consider the subvariety  $\mathfrak{B}$  of variety  $\mathfrak{N}_{(2,3^k)}$  defined by identity  $x^3 \cdot [x_1, x_2, x_3] \dots [x_{3n-2}, x_{3n-1}, x_{3n}] = 1$  and let  $F(\mathfrak{B}; x_1, x_2, \dots, x_{3n}) = F(\mathfrak{B})$  be the  $\mathfrak{B}$ -free loop. By [33]  $F(\mathfrak{B}) = F_{3n}(\mathfrak{N}_{(2,3^k)})/H$ , where  $H$  is the verbal subloop of  $F_{3n}(\mathfrak{N}_{(2,3^k)})$  generated by loop word  $x^3 \cdot [x_1, x_2, x_3] \dots [x_{3n-2}, x_{3n-1}, x_{3n}]$ . The subloop  $H$  is normal in  $F_{3n}(\mathfrak{N}_{(2,3^k)})$ . Then  $H \subseteq F_{3n}(\mathfrak{N}_{(2,3^k)})^3 F_{3n}(\mathfrak{N}_{(2,3^k)})'$ .

From the previous reasonings it follows that subloop  $H$  is finite and  $H \subseteq \Phi(F_{3n}(\mathfrak{N}_{(2,3^k)}))$ . Now we have  $F(\mathfrak{B}; x_1, x_2, \dots, x_{3n}) = F_{3n}(\mathfrak{N}_{(2,3^k)}; x_1, x_2, \dots, x_{3n})/H = F_{3n}(\mathfrak{N}_{(2,3^k)}; x_1H, x_2H, \dots, x_{3n}H) = lp(x_1H, x_2H, \dots, x_{3n}H) = lp(x_1, x_2, \dots, x_{3n}, H) \equiv lp(x_1, x_2, \dots, x_{3n}) = F_{3n}(\mathfrak{N}_{(2,3^k)}; x_1, x_2, \dots, x_{3n})$ . The symbols  $x_i$ ,  $i = 1, 2, 3n$  are free generators. Then  $F_n(\mathfrak{B}) \equiv F_n(\mathfrak{N}_{(2,3^k)})$  for all finite number  $n$ .

By [33, Theorem VI.14.1] every variety of algebras is generated by its algebras of finite rang. Hence  $\mathfrak{B} = \mathfrak{N}_{(2,3^k)}$ . From definition of variety  $\mathfrak{N}_{(2,3^k)}$  it follows that for  $k > 1$  not all non-associative loops from  $\mathfrak{B}$  satisfy the identity  $x^3 = 1$ . (If  $k > 1$  then the quasiidentity  $x^{3^{k-1}} = 1 \rightarrow (x, y, z) = 1$  holds in any commutative Moufang loop  $Q$ , as  $Q^3 \subseteq Z(Q)$  by [3]). We consider  $\mathfrak{B} = \mathfrak{N}_{(2,3^k)}$  as quasivariety. Obviously, the quasivariety  $\mathfrak{B} = \mathfrak{N}_{(2,3^k)}$  satisfies the quasiidentity (5) if and only if  $k = 1$ . In such case the quasiidentity (4) holds.

8c<sub>4</sub>). We shall say that the loop  $L$  of the variety  $\mathfrak{M} \subseteq \mathfrak{N}_2$  has the representation  $L = lp(x_1, \dots, x_n \parallel R = 1)$ , in  $\mathfrak{M}$  if  $L \cong F_n(\mathfrak{M}; x_1, \dots, x_n)/\bar{R}$ , where  $\bar{R}$  is the normal subloop in  $F_n(\mathfrak{M})$  generated by the set  $R \subseteq F_n(\mathfrak{M})$ .

8c<sub>5</sub>). Let  $\mathfrak{N}$  be a variety of commutative Moufang loops and let  $F_n = F_n(x_1, \dots, x_n) = lp(x_1, \dots, x_n)$  denote the  $\mathfrak{N}$ -free loop with free generators  $x_1, \dots, x_n$ . Assume that a loop  $L \in \mathfrak{N}$  has the representation  $L =$

$lp(x_1, \dots, x_n \parallel R = 1)$ , in  $\mathfrak{N}$ , where  $R$  is a set of associator words in  $x_1, \dots, x_n$ . The loop  $F_n$  is finite generated. By [3], [37] the associator subloop  $F'_n$  is finite. Moreover, by [37] the Frattini subloop  $\Phi(F_n)$  is normal in  $F_n$  and  $F'_n \subseteq \Phi(F_n)$ . Then the normal subloop  $\bar{R}$  is finite and  $\bar{R} \subseteq \Phi(F_n)$ . Similar of item 8c3) let  $L \cong F_n(x_1, \dots, x_n)/\bar{R} = lp(x_1, \dots, x_n)/\bar{R} = lp(x_1\bar{R}, \dots, x_n\bar{R}) = lp(x_1, \dots, x_n \cup \bar{R}) = F_n(x_1, \dots, x_n)$  since  $\bar{R} \subseteq \Phi(F_n)$  is a finite loop. Consequently,  $L \cong F_n$ .

8c6). Let  $L = lp(x, x_1, \dots, x_n)$  be a finitely generated commutative Moufang loop and let  $M$  denote the quotient loop of loop  $L$  by the relation  $x^3 = u$ , where  $u \in L'$  is an associator word in variables  $x, x_1, \dots, x_n$ . If  $x \notin \Phi(L)$  then  $x^3 = 1$  in loop  $L$ .

Indeed, from relation  $x^3 = u$  and  $(L')^3 = \{1\}$  [3] it follows that  $x^{3^2} = 1$ . Again by [3],  $x^3 \in Z(L)$  and the relation  $x^3 = u$  can be written in form  $x^3v = 1$  where  $v \in L'$ . Hence  $lp(x^3)$  is a finite normal subloop of  $L$ . As  $L$  is finitely generated then the associant  $L'$  is finite. Then the normal subloop  $R$  of  $L$ , generated by  $x^3v$ , is finite and by item 8c5)  $R \subseteq \Phi(L)$ .

By hypothesis  $M \cong L/R$ . But  $L/R = lp(x, x_1, \dots, x_n)/R = p(xR, x_1R, \dots, x_nR) = p(x, x_1, \dots, x_n, R) = p(x, x_1, \dots, x_n) = L$ . Obviously,  $x^3 = 1$  in  $M$ . Hence  $L = lp(x, x_1, \dots, x_n)$  and  $x^3 = 1$  in  $L$ .

8d). A loop is referred to as monolite if it is finitely generated and is not decomposable in a direct product of two nonunit subloops of it.

8d1). Let  $\mathfrak{M}$  be a variety of commutative Moufang loops. Then any  $\mathfrak{M}$ -free commutative Moufang loop of exponent 0 or  $3^k$  is monolite [43, Proposition 5].

8e). Let  $\mathfrak{M}$  be a quasivariety of commutative Moufang loops and  $L$  be an arbitrary commutative Moufang loop. By [58] (respect. [54]) the least normal subloop  $H$  of  $L$  for which  $L/H \in \mathfrak{M}$  is called the quasiverbal subloop of the loop  $L$  corresponding to the quasivariety  $\mathfrak{M}$  (respect.  $\mathfrak{M}$ -replica of  $L$ ).

8e1). According to [33, pag. 297, 371] (respect. [33, Corollaries V.11.7, V.12.10]) it is possible to consider that the loop  $L$  considered in item 8e) is a free loop of quasivariety  $\mathfrak{M}$ .

8f). **Lemma 1** [54], [55]. *Let  $\mathfrak{M}$  be a non-associative quasivariety from the arbitrary variety  $\mathfrak{N} \in \{\mathfrak{N}_2, \mathfrak{N}_{2,3^k}; k = 1, 2, \dots\}$  and let a loops  $A, B$  of  $\mathfrak{M}$  be represented in  $\mathfrak{N}$  by*

$$A = lp(x_1, \dots, x_n \parallel M(x_1, \dots, x_n) = 1),$$

$$B = lp(y_1, \dots, y_m \parallel N(y_1, \dots, y_m) = 1),$$



where  $M, N$  are totalities of associator words. If  $H$  is the normal subloop of  $C = A \star B$  generated by some associators of the form  $[x_i, x_j, y_k]$  or  $[x_i, y_j, y_k]$ , then  $C/H \in \mathfrak{M}$ .

8f<sub>1</sub>). **Proof.** ... The elements of  $C$  which are not in the subloop  $[A, A, B][A, B, B]$  are approximated by the loop  $A \times B$  via the natural morphism  $\varphi : C \rightarrow C/[A, A, B][A, B, B]$ . By the condition imposed on the generators of  $H$  it follows that  $\ker\varphi = [A, A, B][A, B, B]$ .

8f<sub>2</sub>). Then the loop morphism which we are looking for is the epimorphism  $\varphi : C \rightarrow F_3(\mathfrak{N}; x, y, z) \in \mathfrak{M}$  defined by equalities

$$\begin{aligned} x_p^\varphi = x, \quad x_q^\varphi = x, \quad y_r^\varphi = z, \quad x_i^\varphi = 1(i \neq p, i \neq l, 1 \leq i \leq n), \\ y_j^\varphi = 1, \quad (j \neq r, 1 \leq j \leq m). \end{aligned}$$

8f<sub>3</sub>). According to definition of free product of algebras the loop  $C = A \star B$  defined in item 8f) is defined not by generators  $x_1, \dots, x_n, y_1, \dots, y_m$  of loops  $A$  and  $B$  but is generated by elements of loops  $A$  and  $B$ . Hence, the elements of  $A \star B$  are loop words with respect to elements from  $A \cup B$ . In such a case the elements from  $A \star B$  written down in normal (canonic, non-reduced, least length with respect to generators from  $A \cup B$ ) form can not contain subwords of kind  $uv$ , where  $u, v \in A$  or  $u, v \in B$ . Hence the associators considered in item 8f) have no normal, canonical form.

8f<sub>4</sub>). The contents of item 8f<sub>1</sub>) is a mechanical replication from group theory, using the methods described in item 7a<sub>2</sub>). To hide it the author Ursu V. I. adds the senseless proposition: by the condition imposed on the generators of  $H$  it follows that  $\ker\varphi = [A, A, B][A, B, B]$ . The last equality is not obvious, it is necessary to prove. Here it is received by mechanical replication from group theory. This is confirmed by the following quote from group theory [29, pag.459].

8f<sub>5</sub>). Exists an epimorphism of free product  $\prod_{\alpha \in N}^* A_\alpha$  on direct product  $\prod_{\alpha \in N} A_\alpha$  the same groups, defined by identical mappings of groups  $A_\alpha$ ,  $\alpha \in I$ , on itself. A kernel  $D$  of it epimorphism is called the cartesian subgroup and consists of those and only of those words, at which the product of all factors, belonging to group  $A_\alpha$ , is equal to 1 for all  $\alpha \in I$ . On the other hand, the cartesian subgroup  $D$  is a normal subgroup generated by all commutators of kind  $[a_\alpha, a_\beta]$ , where  $\alpha \neq \beta$  and  $a_\alpha, a_\beta$  belong to the given systems of generators of groups  $A_\alpha, A_\beta$ .

8f<sub>6</sub>). Similar of item 7c) the assertion from item 8f<sub>2</sub>) is false, is a roughest mistake.

8f<sub>7</sub>). According to items 8f<sub>4</sub>) and 8f<sub>6</sub>) we conclude that the proof of Lemma 1 from item 8f) is erroneous, is a complete senseless.

8g). **Lemma 2** [54]. *Let  $\mathfrak{N}$  be one of the quasivarieties  $\mathfrak{N}_2$  or  $\mathfrak{N}_{2,3^k}$ ,  $k \geq 1$ . If a commutative Moufang loop  $L_n$  of nilpotence class 2 has the representation in  $\mathfrak{N}_2$*

$$L_n = lp(x_1, \dots, x_{3n} \parallel \prod_{i=1}^{3n-2} [x_i, x_{i=1}, x_{i=2}] = 1),$$

*then the  $\mathfrak{N}$ -replica of  $L_n$  belongs to the quasivariety  $Q(F_3(\mathfrak{N}))$ .*

8g<sub>1</sub>). The proof of Lemma 2 uses essentially Lemma 1 and the similar assertion from item 8f<sub>2</sub>). From items 8f<sub>6</sub>) and 8f<sub>7</sub>) it follows that the proof of Lemma 2 from item 8g) is erroneous, is completely senseless.

8g<sub>2</sub>). In Lemma 2 the notion of  $\mathfrak{N}$ -replica can be replaced by notion of  $\mathfrak{N}$ -free loop according to item 8e<sub>1</sub>). In such a case it is not necessary to use Lemma 1 and the proof of Lemma 2 becomes obvious.

8g<sub>3</sub>). Lemma 4 is easily proved if using the assertion from item 8c<sub>5</sub>).

8h). **Lemma 5** [54]. *Let  $L$  be a monolithic commutative Moufang loop of nilpotence class 2. Then  $L \in \mathfrak{N}_{(2,3^k)}$  (respectively  $L \in \mathfrak{N}_{(2)}$ ) iff  $L$  satisfies the both following conditions: the exponent of  $L$  is  $3^k$  (respectively is 0), and the set of defining relations of  $L$  in  $\mathfrak{N}_{(2,3^k)}$  (respectively  $\in \mathfrak{N}_{(2)}$ ) consists only of associator words which are equalized to 1.*

8h<sub>1</sub>). The conditions of Lemma 5 are erroneous, senseless. Taking into account items 8c<sub>5</sub>), 8d<sub>1</sub>) and Proposition from item 8c<sub>3</sub>) the correct Lemma 5 is the Proposition 5 from [42]: if a non-associative relatively free commutative Moufang loop has the exponent  $3^k$  or 0 that it is not decomposable in direct product of its subloops.

8h<sub>2</sub>). Unlike the proof of Proposition 5 from [42], presented in item 8h<sub>1</sub>), the proof of Lemma 5 from item 8h) is a totality of erroneous statements in large volume. To reduce significantly the volume of proof let us use items 8c<sub>5</sub>), 8d<sub>1</sub>) and Proposition from item 8c<sub>3</sub>).

8i). The main results of paper [54], the Theorems 1, 2, the Corollaries 1 – 3, similarly assume the groups's assertions from Budkin's and Fedorov's papers [20], [9] of groups, particularly, of nilpotent groups of class 2. [54] does not state it. The proofs are a mechanical replication of group results by the methods described in item 7a<sub>2</sub>) (for example, using the notions of compactness theorem [23], servant subgroups, isolator [7] and others). Reading the work [54], there is an impression that some assertions belong to the author of [54], but actually they are a literal replication of group results.

8i<sub>1</sub>). The contents of the main results of [54] mentioned in item 8i) are erroneous. Their lengthy proofs are also erroneous, using unproved assertions for loops for which the group's analogue are proved in Budkin's and Fedorov's papers [20], [9].

8i<sub>1</sub>). It is possible to correct and prove the main results of [54] by the methods described in item 8h<sub>2</sub>). This decreases essentially the volume of proofs.

8j). According to the above stated we conclude that all results and their proofs from work [54] are erroneous, the work not is mathematical.

9a). Section 1 of [58] contains the used denotations and preliminary results. As mentioned in item 8a<sub>1</sub>), works [58], [59] use the same denotations as [54], see items 8b), 8b<sub>2</sub>), 8c) – 8c<sub>2</sub>), 8c<sub>4</sub>), 8d) – 8e).

9a<sub>1</sub>). In [58] the following notations are also introduced.

$$H_{\infty\infty\infty} = F_3(x, y, z), H_{r\infty\infty} = lp(x, y, z) \parallel x^{3^r} = 1,$$

$$H_{rs\infty} = lp(x, y, z) \parallel x^{3^r} = y^{3^s} = 1,$$

$$H_{rst} = lp(x, y, z) \parallel x^{3^r} = y^{3^s} = z^{3^t} = 1, H_{00t} = Z_{3^t}, H_{00\infty} = Z, H_{000} = \{1\},$$

where  $r, s, t$  are integers such that  $0 \leq r \leq s \leq t$ ;  $H_{r\infty\infty}, H_{rst}, Z_{p^m}$ .

$A_{mk}$  (respectively,  $A_m$ ) =  $lp(a_{ij}, 1 \leq i \leq m, 1 \leq j \leq 3m + 3)$  is a loop of the variety  $\mathfrak{N}_{2,3^k}$  (respectively  $\mathfrak{N}_2$ ) whose determinant relations are

$$\prod_{i=1}^{m+1} [a_{13i-2}, a_{13i-1}, a_{13i}] = \dots = \prod_{i=1}^{m+1} [a_{m3i-2}, a_{1mi-1}, a_{m3i}]. \quad (4)$$

$$[a_{ij}, a_{kl}, a_{pr}] = 1, i \neq k \vee i \neq p \vee k \neq p, 3 < j, l, r < 3m + 3; \quad (5)$$

$B_{lmk} = A_{mk}^1 \times \dots \times A_{mk}^l$  (respectively,  $B_{lm} = A_m^1 \times \dots \times A_m^l$ ) is the Cartesian product of  $l$  copies of the loop  $A_{mk}$  (respectively,  $A_m$ ).

$$a = \prod_{i=1}^{m+1} [a_{13i-2}, a_{13i-1}, a_{13i}] \quad (6)$$

is a element of loop  $A_{mk}$  (respectively,  $A_m$ ).

9a<sub>2</sub>). According to item 8c<sub>5</sub>) the loop  $A_{mk}$  (respectively,  $A_m$ ) defined by associator relations (4) (respectively, (5)) is a  $\mathfrak{N}_{2,3}$ -loop (respectively,  $\mathfrak{N}_2$ -loop).

9a<sub>3</sub>). The auxiliary preliminary results are the Lemmas 1.1 – 1.6.

9a<sub>4</sub>). The Lemma 1.2 is the false Lemma 1 from item 8f), see the items 8f<sub>1</sub>) – 8f<sub>7</sub>).

Lemma 1.3 [54].  $F_{3n}(\mathfrak{N}_{(2,3^k)})$  (resp.,  $\mathfrak{N}_{(2)}; x_1, \dots, x_{3n})/lp([x_1, x_2, x_3] [x_4, x_5, x_6] \dots [x_{3n-2}, x_{3n-1}, x_{3n}]) \in T_k$  (resp.,  $T$ .)

The Lemma 1.3 is given as a result of [54], but in such a form it is not present in [54]. Actually Lemma 1.3 is a mechanical replacement from

the group's theory as is shown in item 7a<sub>2</sub>). See also the items 8h) – 8h<sub>2</sub>) regarding Lemma 1.3.

9b. As was shown in [59], the subquasivarieties of  $\mathfrak{N}_{2,3}$  are characterized by associator quasivarieties. For  $k \geq 2$ , we shall emphasize in  $\mathfrak{N}_2$  and in  $\mathfrak{N}_{2,3^k}$ , those quasivarieties which have the same structure and are also characterized by associator quasivarieties.

9b<sub>1</sub>. The assertions from item 9b are false by item 8c<sub>5</sub>).

9c). Lemma 1.5. *For any  $l \geq 1$ , the lattices of the non-associative quasivarieties  $\mathfrak{N}_{(2,3^l)}$  and  $\mathfrak{N}_{(2)}$  are isomorphic.*

9c<sub>1</sub>). Proof. According to the quasi-identities (4), (5) (respectively (5)–(7)) of item 8c<sub>2</sub>) the determinant relations of any monolite non-associative loop of  $\mathfrak{N}_{(2,3^l)}$  (respectively,  $\mathfrak{N}_{(2)}$ ) are equalities of some associator words to unit elements (see Lemma 5 of [58]).

9c<sub>2</sub>). It is clear that the non-associative subquasivarieties of the quasivarieties  $\mathfrak{N}_{(2,3^l)}$  and  $\mathfrak{N}_{(2)}$  are generated by monolite loops.

9c<sub>3</sub>). We establish a reciprocal correspondence between non-associative monolite loops of  $\mathfrak{N}_{(2,3^l)}$  and monolite non-associative loops of  $\mathfrak{N}_{(2)}$  in the following way. We consider that to the monolite non-associative loop  $A^l \in \mathfrak{N}_{(2,3^l)}$  with generators  $a_1^l, \dots, a_n^l$  and the determinant relations in  $\mathfrak{N}_{(2,3^l)}$   $R(a_1^l, \dots, a_n^l) = 1$ , there corresponds the monolite non-associative loop  $A \in \mathfrak{N}_{(2)}$  with generators  $a_1, \dots, a_n$  and determinant relations (in  $\mathfrak{N}_{(2)}$ )  $R(a_1, \dots, a_n) = 1$ .

9c<sub>4</sub>). Lemma 1.5 is false. According to Proposition of item 8c<sub>3</sub>) it is necessary to consider the case  $l = 1$ .

9c<sub>5</sub>). It is necessary to consider the free loops instead of monolite loops according to item 8f<sub>5</sub>) in items 9c<sub>2</sub>), 9c<sub>3</sub>). Even in such a case the assertion from item 9c<sub>2</sub>) is non-correct. By [33] the quasivarieties are not always characterized by free algebras.

9c<sub>6</sub>). According to items 9c<sub>4</sub>) and 8c<sub>5</sub>) the assertion from item 9c<sub>3</sub>) is senseless. The proof of Lemma 1.5 is very lengthy. Similarly to items 9c<sub>1</sub>) – 9c<sub>5</sub>), a part of the proof is senseless. Consequently, Lemma 1.5 and its proof is false, is senseless.

9d). Lemma 1.6. *For  $m = 1, 2, \dots$  we have  $A_{mk} \in T_k$ ,  $A_m \in T$ .*

9d<sub>1</sub>). Proof. An element  $a \in A_{mk}$  (respectively,  $A_m$ ) is approximated by the morphism of loops  $\varphi : A_{mk} \rightarrow F_3(\mathfrak{N}_{2,3^k}; x, y, z)$  (respectively,  $A_m \rightarrow F_3(x, y, z)$ ) defined by the equalities ...

9d<sub>2</sub>). The proof of Lemma 1.6 is false as in item 9d<sub>1</sub>) a similar mistake is made as in item 7c) and the false Lemma 1.2 from item 9a<sub>4</sub>) is used.

9e). The work [59] is a direct continuation of work [58] and all assertions

of [58], in particular, Sections 2 and 3, are intended to prove the main result of [59], the Theorem from item 10a<sub>1</sub>). Now let us analyze in detail Sections 2, 3 and also [59] due to its large volume. Note that [58], [59] are not mathematical works. All results of these works are erroneous, [58], [59] are completely senseless. We will confirm this below only partially.

9e<sub>1</sub>). The false Lemmas 1.2 – 1.6 from [58], considered earlier, are applied repeatedly.

9e<sub>2</sub>). The false reasonings, described in item 7c) with respect to existence of homomorphisms loops, similar to Dick's Theorem, are applied repeatedly.

9f). Following [58, pag. 40] we define the loop  $H_m$  of the variety  $\mathfrak{N}_{2,3k}$  (respectively,  $\mathfrak{N}_2$ ), which is essentially investigated in Section 2, as follows:

9f<sub>1</sub>).  $B = B(n, k)$  (respectively,  $B(n) = lp(x, x_1, \dots, x_{3n} \parallel [x, x_1^{\alpha_1}, \dots, x_{3n}^{\alpha_{3n}}, x_1^{\beta_1}, \dots, x_{3n}^{\beta_{3n}}] = 1)$ );

9f<sub>2</sub>).  $H = H(n, k)$  (respectively,  $H(n)$ ) is the factor-loop of the loop  $B$  by the relation  $x^3 = \prod_{i=1}^n [x_{3i-2}, x_{3i-1}, x_{3i}]$ ;

9f<sub>3</sub>).  $H_m$  is the factor-loop of the  $\mathfrak{N}_2$ -free product of the loops  $B$  and  $A_{mk}$  (respectively,  $A_m$ ) by the relation  $x^3 = \prod_{i=1}^n [x_{3i-2}, x_{3i-1}, x_{3i}]a$ , where  $a$  is defined by (6) of item 9a<sub>1</sub>).

9f<sub>4</sub>). Every  $n$ -generated commutative Moufang loop is centrally nilpotent by Bruck-Slaby's Theorem [3]. Then the equality  $[x, x_1^{\alpha_1}, \dots, x_{3n}^{\alpha_{3n}}, x_1^{\beta_1}, \dots, x_{3n}^{\beta_{3n}}] = 1$  from item 9f<sub>1</sub>) holds in any commutative Moufang loop. Then the loop  $B(n, k)$  (respectively,  $B(n)$ ) defined in item 9f<sub>1</sub>) is the  $\mathfrak{N}_{2,3k}$ -free (respectively,  $\mathfrak{N}_2$ -free) loop with free generators  $x, x_1, \dots, x_{3n}$ . The last statement follows also from item 8c<sub>5</sub>).

9f<sub>5</sub>). By item 8c<sub>6</sub>) the loop  $H(n, k)$  (respectively,  $H(n)$ ) is generated by elements  $x, x_1, \dots, x_{3n}$  and  $x^3 = 1$ . Then from item 9f<sub>4</sub>) it follows that  $H(n, k)$  (respectively,  $H(n)$ ) is the  $\mathfrak{N}_{2,3}$ -free loop with free generators  $x, x_1, \dots, x_{3n}$ .

9f<sub>6</sub>). In relation to item 9f<sub>3</sub>) it follows that  $x^3 = 1$  by item 9f<sub>4</sub>) or 8c<sub>6</sub>). Then  $\prod_{i=1}^n [x_{3i-2}, x_{3i-1}, x_{3i}]a = 1$ . From here it follows that  $B = A_{mk}$  or  $\prod_{i=1}^n [x_{3i-2}, x_{3i-1}, x_{3i}] = 1$  and  $\prod_{i=1}^{m+1} [a_{3i-2}, a_{3i-1}, a_{3i}] = 1$ .

9f<sub>7</sub>). The items 9f<sub>5</sub>), 9f<sub>6</sub>) contradict all researches of Section 2 of [58] and of the main result of paper [59], the Theorem of item 10a<sub>1</sub>). We confirm it on the example of Lemma 2.1.

9g). Lemma 2.1.  $H_m \in Q(H)$ .

9g<sub>1</sub>). Proof. An element  $a \in H_m$  is approximated by the loop  $F_3(\mathfrak{N}_{2,3k}; u, v, w)$  (respectively,  $F_3(u, v, w)$ ) via the morphism of loops  $\varphi : H_m \rightarrow F_3(\mathfrak{N}_{2,3k})$  (respectively,  $H_m \rightarrow F_s$ ) defined by the equations:

$$x_1^\varphi = u^{-1}, x_2^\varphi = v, x_3^\varphi = w, x_4^\varphi = 1, \dots, x_{3n}^\varphi = 1, x^\varphi = 1,$$

$$\begin{aligned}
a_{j_1}^\varphi &= u, a_{j_2}^\varphi = v, a_{j_3}^\varphi = w, i = 1, \dots, m; \\
a_{i_j}^\varphi &= 1, i = 1, \dots, m, j = 4, \dots, 3m + 3.
\end{aligned}$$

9g<sub>2</sub>). Now, we show that  $H_m/lp(a) \in Q(H)$ .

9g<sub>3</sub>). Indeed,  $H_m/lp(a) \equiv H \star (A_{mk}/lp(a))$  (respectively,  $H \star (A_m/lp(a))$ ), where  $A_{mk}/lp(a) \in T_k$  (respectively,  $A_m/lp(a) \in T$ ). But  $T_k \in Q(H)$  (respectively,  $T \in Q(H)$ ).

9g<sub>4</sub>). Then by Lemma 1.2, we have  $H_m/lp(a) \in Q(H)$ . The proof is complete.

9g<sub>4</sub>). According to item 7c) the definition of morphism of loops  $\varphi : H_m \rightarrow F_3(\mathfrak{N}_{2,3^k})$  (respectively,  $H_m \rightarrow F_s$ ) from item 9g<sub>1</sub>) is false.

9g<sub>5</sub>). The assertion from item 9g<sub>2</sub>) is false. The subloop  $lp(a)$  is not normal in loop  $H_m$ .

9g<sub>6</sub>). Similarly to item 9g<sub>5</sub>) the expression  $A_{mk}/lp(a)$  is false. The expression  $T_k \in Q(H)$  from item 9g<sub>3</sub>) is also false.

9g<sub>7</sub>). According to item 9g<sub>6</sub>) the Lemma 1.2 mentioned in item 9g<sub>4</sub>) is false.

9g<sub>8</sub>). According to items 9g<sub>4</sub>) – 9g<sub>7</sub>) we conclude that the Lemma 2.1 from item 9g) is completely senseless. Moreover, all results of Section 2 are false.

9h). All results of Section 3 are a research of loop  $L'_m$ . Let us define it. But at first we remind that item 9a<sub>1</sub>) defines the loop  $B_{lmk}$ . Take the loop  $B_{l_1n_1} = A_{m_1}^{l_1i} \times \dots \times A_{m_1}^{l_1i}$  and denote the generators of the subloop  $A_{m_1}^j$ ,  $1 \leq j \leq l_i$ , by  $a_{11}^{ji}, a_{12}^{ji}, \dots, a_{m,3m+3}^{ji}$ , respectively. Now, define the loop  $L'_m$  as the factor-loop of the  $\mathfrak{N}_{2,3}$ -free product of the loops  $B_{l_1n_1}$  and  $F_n(\mathfrak{N}_{2,3}; z_1, \dots, z_{ni})$  by the relations

$$H_i(z_1, \dots, z_{ni}) = 1, \quad (11)$$

$$[a_{rs}^{ji}, a_{pq}^{ki}, z_{li} = 1(j = k \rightarrow \notin \{3j - 2, 3j - 1, 3j\}), \quad (12)$$

$$[a_{rs}^{ji}, z_{pi}, z_{qi}] = 1(p \vee q \notin \{3j - 2, 3j - 1, 3j\}), \quad (13)$$

$$a^{1i}[z_{1i}, z_{2i}, z_{3i}]v_{1i} = \dots = a^{i1i}[z_{3l_i-2,i}, z_{3l_i-1,i}, z_{3l_i,i}]v_{li}, \quad (14)$$

where  $a^{1i}, \dots, a^{li}$  are elements of the loops  $A_m^{1i}, \dots, A_{ml}^{li}$ .

9h<sub>2</sub>). Lemma 3.1.  $L'_m \notin T_1$ .

9h<sub>3</sub>). Proof. Let  $\varphi : L'_m \rightarrow F_3(\mathfrak{N}_{2,3})$  be a morphism of loops.

9h<sub>4</sub>). The definition of loop  $L'_m$  uses essentially Lemma 1.2 from item 8f). Then from relations (12) – (14) of item 9h) and item 9a<sub>4</sub>) it follows that the definition of loop  $L'_m$  is false, not trivial loops  $L'_m$  do not exist.

9h<sub>5</sub>). The assertion of item 9h<sub>3</sub>) is false by item 7c).

9h<sub>6</sub>). Lemma 3.1 from item 9h<sub>2</sub>) is false according to items 9h<sub>4</sub>), 9h<sub>5</sub>).

9h<sub>7</sub>). The definition of loop  $L'_m$  from item 9h) is almost literally stated in paper [58]. It consists of plenty of symbols with confusing indexes which do not correspond to the previously considered indexes.

It is impossible to read. The entire Section 3 is written in the same manner. From Section 3 we cited only items 9h<sub>2</sub>), 9h<sub>3</sub>). But it is enough to conclude.

9h<sub>8</sub>). According to items 9h<sub>4</sub>) – 9h<sub>7</sub>) we conclude that Section 3 consists only of senseless and false notions and results.

10a). Now let us analyze the main result of papers [58], [59], the Theorem from [59]. We cite items 10a<sub>1</sub>) – 10a<sub>7</sub>).

10a<sub>1</sub>). Theorem. A lattice  $\mathcal{K}$  of subquasivarieties of commutative Moufang loops with nilpotency class  $\leq 2$  is finite if and only if any quasivariety in  $\mathcal{K}$  is generated by a finite set of loops of the types  $H_{\infty\infty\infty}$ ,  $H_{r\infty\infty}$ ,  $H_{rs\infty}$ ,  $H_{rst}$ ,  $Z_{p^m}$ , where  $p \neq 3$  is a prime number.

10a<sub>2</sub>). Proof. Sufficiency. ... According to the belonging criteria, this will be true if every finitely generated loop  $B \in \mathcal{K}$  is approximated by its subloops generated by three elements.

We show that it is indeed the case. If  $b \in B$  and  $b \notin B'$  then, obviously,  $b$  is approximated by a cyclic subgroup.

10a<sub>3</sub>). Let  $b \in B'$ . Since  $H_{\infty\infty\infty}$ ,  $H_{r\infty\infty}$ ,  $H_{rs\infty}$ ,  $H_{rst}$  are approximated respectively by the loops  $Z_{3^t} \times F_3(N_{2,3})$ ,  $Z_{3^t} \times Z \times F_3(N_{2,3})$ ,  $Z_{3^t} \times Z \times F_3(N_{2,3})$ ,  $Z \times F_3(N_{2,3})$ ,  $b$  is approximated by the loop  $F_3(N_{2,3})$ .

10a<sub>4</sub>). Let  $\varphi : B \rightarrow F_3(N_{2,3}; x, y, z)$  be a morphism of loops such that  $b^\varphi = [x, y, z]$ .

10a<sub>5</sub>). Necessity. Let  $\Sigma$  be the set of all loops of the types indicated in item 9a<sub>1</sub>) and let  $\mathfrak{K}$  be a quasivariety which contains only a finite number of subquasigroups. Suppose for a contradiction that  $\mathfrak{K}$  is not generated by a finite set of finite nonisomorphic loops of  $\Sigma$ .

Let  $L \in \mathfrak{K}$  be a finite commutative Moufang loop of exponent equal to a power of 3. For some elements  $a, b_1, \dots, b_{3n} \in L$  let  $a^3 = \prod_{i=1}^n [b_{3n-2}, b_{3n-1}, b_{3n}] \neq 1$ .

10a<sub>6</sub>). Let  $[a, b_1^{\alpha_1} \dots b_{3n}^{\alpha_{3n}}, b_1^{\beta_1} \dots b_{3n}^{\beta_{3n}}] = 1$ , where  $0 \leq \alpha_i < 3$ ,  $0 \leq \beta_i < 3$ . Let us investigate the commutative Moufang loop  $B = lp(x, y_1, \dots, y_{3n} \parallel x^3 = \prod_{i=1}^n [y_{3n-2}, y_{3n-1}, y_{3n}], x, y_1^{\alpha_1} \dots y_{3n}^{\alpha_{3n}}, y_1^{\beta_1} \dots y_{3n}^{\beta_{3n}}] = 1)$ , where  $3^k$  (respectively 0) is the exponent of the loop  $L$  (and the relations are given in the variety  $\mathfrak{N}_{2,3k}$  (respectively,  $\mathfrak{N}_2$ )). We show that the loop  $B \in Q(L)$ .

10a<sub>7</sub>). Denote  $T = lp(x^3, [x, y, z]$  for all elements  $y, z \in B) \subset B$ . Any element of the subloop  $T$  is approximated by  $L$  via the morphism of loops

of  $B$  in  $L$ .

10a<sub>8</sub>). The assertion from item 10a<sub>3</sub>) are not obvious, they need to be proved. Actually this statement is false.

10a<sub>9</sub>). The definition of morphism  $\varphi : B \rightarrow F_3(N_{2,3}; x, y, z)$  from item 10a<sub>4</sub>) contradicts item 7c).

10a<sub>10</sub>). From item 8c<sub>6</sub>) it follows that  $a^3 = 1$  for element  $a$  defined in item 10a<sub>5</sub>).

10a<sub>11</sub>). According to item 9f<sub>4</sub>) the loop  $B$  from item 10a<sub>6</sub>) is free. Then the assertion from item 10a<sub>7</sub>) is false.

10a<sub>12</sub>). The proof of necessity uses essentially the false Lemmas 1.3, 1.5, 2.6, 3.16 from [58].

10a<sub>13</sub>). From items 10a<sub>8</sub>) – 10a<sub>12</sub>) it follows that the proof of the main result of paper [58], [59], the Theorem from item 10a<sub>1</sub>), is not proved, is completely senseless.. Moreover, the contents of Theorem is false. It confirms also the following statement.

10b). The definitions of loops  $H_{r\infty\infty}$ ,  $H_{rs\infty}$ ,  $H_{rst}$  from item 10a<sub>1</sub>) make sense if  $r = s = t = 1$ . In such a case they are free commutative Moufang loops with three free generators satisfying identity  $x^3 = 1$ .

10b<sub>1</sub>). Indeed, remind that a loop  $L$  of a variety  $\mathfrak{M}$  has the representation  $L = lp(x_1, \dots, x_n \parallel R = 1)$  in  $\mathfrak{M}$  if  $L \cong F_n(x_1, \dots, x_n)/\bar{R}$ , where  $\bar{R}$  is the normal subloop of  $\mathfrak{M}$ -free loop  $F_n$  with free generators  $x_1, \dots, x_n$ , generated by the set  $R \subseteq F_n$ .

We consider the loop  $H_{r\infty\infty} = lp(x, y, z \parallel x^{3^r} = 1)$ . Then  $H_{r\infty\infty} \equiv F_3(x, y, z)/\bar{(R)}$ , where  $\bar{(R)}$  is the subloop of  $F_3(x, y, z)$  generated by  $x^{3^r}$ . By item 8c<sub>3</sub>)  $(R) \subseteq \Phi(F_3)$ . Then from here Similarly of item 8c<sub>6</sub>) it follows that  $r = 1$ .

Hence,  $x^3 = 1$  in the free loop  $F_3(x, y, z)$ . In every free loop every one-to-one mapping of free generators on free generators proceeds up to automorphism. Then from  $x^3 = 1$  it follows that  $y^3 = z^3 = 1$ .

10c). Further, all researches of Section 2 from [59] are connected with loops  $H_{rst}$ , namely with condition  $r, s, t > 1$ . From items 10b), 10b<sub>1</sub>) it follows that all these researches are without sense.

10d). According to items 10a<sub>13</sub>), 10c) we conclude that paper [59], as well as [58], consists mainly of false, senseless statements.

Now let us analyze papers [64] – [68]. Similarly to [16] (see items 2a) – 2n)) These papers are published in the works of various conferences to avoid reviews.

11a). Let us start the analyses of paper [64]. Section 2 introduce the



notions of centrally nilpotent and centrally solvable loops in a slightly papers [56], [60], [61], [66] – [68], see items  $1a_3$ ,  $2a_1 - 2b_2$ ,  $5b_2$ ,  $6a$ ). As mentioned before, this was done to hide plagiarism. Actually the notions and results of Sections 2 and 3. It is stated in detail in [14, Sections 2, 3].

11b). Now we quote the begin of Section 4.

Next we shall show that the following assertion is trues: *a) the subloops, b) the factor loops and c) the direct product of a finite number of finitely separable loops are also finitely separable.*

11b<sub>1</sub>). The first paragraph after item 11b) contains the proofs of statements a) and b) of item 11b). But these proofs are a plagiarism. The proof of Theorem 3 from [32] for algebras with permutable congruences literally corresponds for particular case, for loops.

11b<sub>2</sub>). The statement c) of item 11b) is proved in the following paragraph. The proof given here literally coincides with proof of Theorem 4 from [32]. The only distinction between these proofs: word "group" was replaced by varies on the word "loop", record " $ab^{-1}$ " in group varies by record " $a/b$ " for loop.

11c). Further the following comes.

Let us investigate now an example where we indicate an finitely approximable nonseparable loop. In the variety of commutative nilpotent of class 2 Moufang loops we consider the free loop  $F$  of strictly countable rank and the loop  $L$  given by the generators  $x, x_i, i = 1, 2, \dots$ , and the relations

$$x_i^3 = 1, [x_i, x_{2i}, x_{3i}] = x, i = 1, 2, \dots,$$

$$[x_j, x_k, x_l] = 1, (j, k, l) \notin \{(i, 2i, 3i) | i = 1, 2, \dots\}.$$

This loop  $L$  has the associant-commutant  $H = \{1, x, x^2\}$ , which coincides with its center.

11c<sub>1</sub>). The example from item 11c) is false. More particularly, the related example is a complete nonsense. We explain it. The commutative Moufang loop  $L$  is centrally nilpotent of class 2, then  $L' \neq L$ . From relations  $L^3L' = \Phi(L)$  (item 8c<sub>3</sub>),  $L' = \Phi(L)$  and  $[x_i, x_{2i}, x_{3i}] = x$  it follows that  $x \in \Phi(L)$ . But  $x$  is a generating element. We get a contradiction.

11d). Further, similarly to items 11b<sub>1</sub>), 11b<sub>2</sub>), something improbable follows. The author V. Ursu literally copies, with one insignificant change (see item 11f)) almost the whole paper of A. I. Mal'cev [32]. The only specific thins of paper [32] is item 11e).

11d<sub>1</sub>). Dear author V. Ursu, the name of A. I. Mal'cev is a sacred name in mathematics. To assume to yourself the ideas and results of A. I. Mal'cev

is not like assuming to yourself the notions and results of N. Sandu, A. Budkin and others. It is a disrespect to A. I. Mal'cev, it is a crime.

11e). A.I. Mal'cev in [32] studied limited solvable groups, and proved that a limited solvable group is finitely separable. Next we will show that this result can be extended to limited solvable Moufang loops.

11f). I think that in an attempt to hide plagiarism Lemma 7 was used instead of Lemma 2 from [32], as it has the same meaning as Lemma 2. It is simple to prove that Lemma 7 is similar to Lemma 2.

11f<sub>1</sub>). To item 11f) refers also to the following quotes from the Introduction.

11f<sub>2</sub>). In this paper we study in detail the structure of finite separable solvable Moufang loops. In particular, it appears that solvable Moufang loops with maximum condition for subloops finite separable, which amplifies results from the theory of residual finite theory ([1] Hirsch, K.A., On infinite solvable groups, J. London Math. Soc. 27 (1952), 81-85, [2] Grünberg, K.W., Residual properties on infinite solvable groups, Proc. London Math. Soc. 3(7) (1957), 29-62.).

11f<sub>3</sub>). It turned out that in a Hausdorff topology all subloops of finite separable solvable Moufang loop are closed sets. At the end prove the existence of an algorithm to solve the membership problem for nilpotent Moufang loops (commutative Moufang loops or nilpotent  $A$ -loops).

11g). Now we list the achievement of V. Ursu. Section 4. If one wants to read the Mal'cev paper [32] it is enough to read the present analyzed Ursu's paper [64] taking into account the following: a) to change designation of Lemma 7 by Lemma 2 (see item 11f)); b) the designation  $L'$  for associant-commutant by  $L_1$  for commutant. Then:

the quote after Definition 1 is the beginning of Section 5;

Lemma 6 is Lemma 1;

Lemma 8 is Lemma 3;

Corollary 1 is the quote after Lemma 3;

Theorem 1 is Theorem 6;

Corollary 2 is the quote after Theorem 6.

11h). In the end of Section 4 V. Ursu assumes again to himself another's assertions: every finitely generated centrally nilpotent Moufang loop or  $A$ -loop satisfies the maximum condition for subloops. The author was notified on the inaccuracy of his proofs in [56], [61], see items 1g<sub>2</sub>), 2l). Moreover, V. Ursu was informed about the authorship of these assertions, see, for example, [14].

11i). Section 5 continues the plagiarism described in items 11b<sub>1</sub>), 11b<sub>2</sub>), 11d), 11g). They literally copied from Section 7 of Mal'cev's paper [32] the

decision of two tasks, mentioned in item 11f<sub>3</sub>). Similarly to item 11d) the Mal'cev's paper [32] is not mentioned.

11i<sub>1</sub>). Section 5 consists of two items a), b). Item b) contains for loops a literal, word for word copy (particularly is identical: the word "algebra" was replaced by word "loop") of the algorithm of solving the problem about membership of element in given subalgebra of finite residual algebra.

11i<sub>2</sub>). **Theorem 3.** *If the nilpotent Moufang loop (commutative Moufang loop or nilpotent A-loop)  $L$  is defined by a finite number of generators and by a finite number of relations, then for the loop  $L$  the membership problem of a subloop element is algorithmic solvable.*

11i<sub>3</sub>). The proof of Theorem 3 from item 11i<sub>2</sub>) uses the positive decision of a problem of words for finitely generated centrally nilpotent Moufang loops or A-loops referring to work [56], [16]. But by items 1h<sub>1</sub>), 2k) the proofs of word problem are erroneous.

11j). It is known [3], [1] that the congruences of groups, of loops one-to-one are characterized with the help of normal subgroups, of normal subloops. Item a) from Section 5 is another plagiarism of V. Ursu, an attempt to transfer the Mal'cev result from Section 6 in [32] about algebra with set of congruences on loop with appropriate set of normal subloops. But here there is a nuance, see item 11l<sub>4</sub>). We explain it. Let's follow [32, Section 6] in items 11k) – 11k<sub>2</sub>) and [64, Section 5, item a)] in items 11l) – 11l<sub>3</sub>), 11m).

11k). For abstract algebra  $A$  with some set  $S$  of its congruences with defined properties is defined the  $S$ -topologies considering open set the coset classes of congruences in  $S$ . Let  $\mathcal{K}$  be a class of algebras of same type as  $A$ . Every homomorphism of  $A$  into algebra in  $\mathcal{K}$  defines a congruence and let  $S$  be the set of such congruences.

11k<sub>1</sub>). We consider that algebra  $A$  is  $\mathcal{K}$ -approximate. Then the  $S$ -topologies on  $A$  is called  $\mathcal{K}$ -topology. It is proved that the algebras with  $\mathcal{K}$ -separable subalgebras are only those algebras for which all abstract subalgebras are closed in  $\mathcal{K}$ -topology.

11k<sub>2</sub>). If  $\mathcal{K}$  is the class of all finite algebras, then  $\mathcal{K}$ -topology is called finite. Then according to definition of finite separable loop the statement from item 11k<sub>1</sub>) is formulated as follows.

11k<sub>3</sub>). *A solvable loop is finite separable if all it abstract subloops are closed in finite topology.*

11l). Particularly, we obtain the following: the finitely separability of  $H$  in relation to the all elements of  $L$  that do not belong to  $H$  is equal to the fact that  $H$  should be closed in finitely topology.

11l<sub>1</sub>). In conclusion the loops with finitely separable subloops are those

loops whose subloops in finite topology are finite sets.

11l<sub>2</sub>). Now according to Theorem 1, we can affirm the following statement.

11l<sub>3</sub>). **Theorem 2.** *All subloops of finite separability solvable Moufang loops are closed sets.*

11l<sub>4</sub>). Comparison of the statement from item 11l<sub>3</sub>) with Theorem 2 from item 11l<sub>3</sub>) shows complete incompetence of author V. Ursu. It confirms the contents of items 11l) – 11l<sub>2</sub>). -The notion of finitely topology from item 11l) is not determined and is not clear. The expression from item 11l<sub>1</sub>) is without sense. The role of Theorem 1 in item 11l<sub>2</sub>) and the role of Moufang loop in item 11l<sub>3</sub>) are not clear.

11l<sub>5</sub>). The role of the text from item 11m) is also not clear. I think that it can be found in enough papers [44], [45].

11m). Finally I want to bring thanks Prof. Mitrofan Chioban for good will to discuss this paper and for valuable comments.

11n). According to the above-stated we conclude that to publish such a composition as [64] is a crime.

12a). Paper [65] deserves the same appraisals as paper [64] from item 11n). This work literally coincides with item a) of Section 5 from [64].

13a). We continue the analysis of works [66] – [68], mentioned in item 11a). Papers [66] and [67] are literally identical.

14a). Let's follow [66]. We quote items 14a<sub>1</sub>) – 14a<sub>5</sub>), 14b) – 14b<sub>3</sub>), 14c) – 14c<sub>8</sub>).

14a<sub>1</sub>). We shall use some notions and results from the monograph of R.H. Bruck [3].

14a<sub>2</sub>). The Moufang Loop (ML) is an  $\langle L, \cdot, {}^{-1} \rangle$  algebra of type  $\{2; 1\}$  whose operations and elements satisfy the following identities:

$$x(y \cdot xz) = (xy \cdot x)z \quad (1)$$

$$x^{-1} \cdot xy = y = yx \cdot x^{-1} \quad (2)$$

where by  $x^{-1}$  we denote the result of the unary operation applied to the  $x$  element.

14a<sub>3</sub>). We observe that the (2) involves the identity  $y \cdot (x^{-1})^{-1} = yx$ , which in its turn involves the identity  $(x^{-1})^{-1} = x$ , that helps to deduce the identity

$$x \cdot x^{-1}y = y = yx^{-1} \cdot x \quad (3)$$

14a<sub>4</sub>). For a certain  $x \in L$  element we denote  $e = x^{-1} \cdot x$ . Then, according to the identities (1) – (3) we will have

$$ye = x^{-1} \cdot x(ye) = x^{-1}[x \cdot y(xx^{-1})] = x^{-1}[(xy \cdot x)x^{-1}] = x^{-1} \cdot xy = y$$

for any  $y \in L$ .

14a<sub>5</sub>). It results that  $e = y^{-1} \cdot y$  and, therefore,  $e$  doesn't depend on the  $x$  element. Then, taking in consideration that (3)

$$e \cdot y = yy^{-1} \cdot y = y$$

for any  $y \in L$ , it follows that  $e$  is a unit element of ML  $L$ .

14a<sub>6</sub>). The definition of Moufang loop from item 14a<sub>2</sub>) differs from definition from the monograph [3], mentioned in item 14a). Moreover, it is false.

14a<sub>7</sub>). It is not clear how identity  $(x^{-1})^{-1} = x$  in item 14a<sub>3</sub>) follows from (2). Hence, identity (3) is not proved.

14a<sub>8</sub>). The identical transformations from item 14a<sub>4</sub>) are not clear, are false.

14a<sub>9</sub>). According to item 14a<sub>8</sub>) the assertion from item 14a<sub>5</sub>) is false.

14b). For  $x, y$  and  $z$  elements from ML  $L$  the associator  $[x, y, z]$  and the commutator  $[x, y]$  are defined by the equalities  $[x, y, z] = (x \cdot yz)^{-1} \cdot (xy \cdot z)$  and  $[x, y] = x^{-1} \cdot y^{-1}(xy)$ , respectively.

The set  $Z(L) = \{x \in L | [x, y, z] = e, [x, y] = e \text{ for any } y, z \in L\}$  is called the ML  $L$  center.

14b<sub>1</sub>). With the help of the induction hypothesis special associator-commutator of the  $n$  multiplicity its defined;  $x_1$  is a special associator-commutator of 1 multiplicity; if  $u$  is a special associator of  $n$  multiplicity which the includes exactly  $i_n$  variables, then  $[u; x_{i_{n+1}}], [u; x_{i_{n+1}}, x_{i_{n+2}}]$  is a special associator-commutator of  $n + 1$  multiplicity.

14b<sub>2</sub>). ML  $L$  is called (central-)nilpotent (NML) of  $n$  class or  $n$ -nilpotent if for any values of the variables in  $L$  the value of any special associator-commutator of  $n + 1$  multiplicity is equal to the  $e \in L$  unity element, but the value of at least one special associator-commutator of  $n$  multiplicity is different from  $e$ .

14b<sub>3</sub>). According to [56] in any nilpotent Moufang loop of class 2 the following identities are true (4) . . . (10).

14b<sub>4</sub>). The definition of center  $Z(L)$  from item 14b) differs from definition in the monograph [3], mentioned in item 14a).

14b<sub>5</sub>). According to item 14b) it is logical to use expression "associator-commutator" instead of "associator-commutator" in item 14b<sub>1</sub>).

14b<sub>6</sub>). The author purposely distors the notions from items 14b<sub>4</sub>) and 14b<sub>5</sub>). It is made to confuse the reader, to hide the truth presented in item 14b<sub>8</sub>).

14b<sub>7</sub>). The notion of special associant-commutator of  $n$  multiplicity in monograph [3] is absent. Moreover, in [3] the notion of centrally nilpotent loop is defined with the help of upper central series.

14b<sub>8</sub>). Actually, author V. Ursu has copied the notions of special associant-commutator and centrally nilpotent loop the and assumed to himself from Sandu's manuscripts of monograph (lost together with the reviews on fault of V. Ursu) and papers, see, for example, [40], [42], [35], [14], [38] and others. These papers, detailed in [14], prove the equivalence of definitions of centrally nilpotent loop from [3], item 14b) and items 2d) – 2e<sub>1</sub>). From Ursu's works follows that the author not understands and is not capable to understand the proof by this equivalence.

14b<sub>9</sub>). According to item 1e<sub>3</sub>) the identities (4) – (10), mentioned in item 14b<sub>3</sub>), are not proved in [56]. In particular, the identity (5)  $[x \cdot y, z, t] = [x, z, t][y, z, t]$ , which is essentially used in proofs from [66], [67], is not proved. This is confirmed by the proof of identity (5.30) from [3, pag. 128].

14c). The following varieties are defined in the class of all 2-nilpotent Moufang loops:

$$\mathcal{K}_{1,0,0} = \text{mod}\{[x, y, z] = e\},$$

$$\mathcal{K}_{1,p,0} = \text{mod}\{[x, y, z] = e, [x, y]^p = e\},$$

$$\mathcal{K}_{1,p,p^m} = \text{mod}\{[x, y, z] = e, [x, y]^p = e, x^{p^m} = e\},$$

where  $m = 2, 3, \dots$  for  $p = 2$  and  $m = 1, 2, \dots$  for any prime number  $p \geq 3$ ,

$$\mathcal{K}_{2,0,0} = \text{mod}\{[x, y, z]^2 = e\},$$

$$\mathcal{K}_{2,2,0} = \text{mod}\{[x, y, z]^2 = e, [x, y]^2 = e\},$$

$$\mathcal{K}_{1,2,2^m} = \text{mod}\{[x, y, z]^2 = e, [x, y]^2 = e, x^{2^m} = e\},$$

$$\mathcal{K}_{3,0,0} = \text{mod}\{[x, y, z]^3 = e\},$$

$$\mathcal{K}_{3,1,0} = \text{mod}\{[x, y, z]^3 = e, [x, y] = e\},$$

$$\mathcal{K}_{3,1,3^m} = \text{mod}\{[x, y, z]^3 = e, [x, y] = e, x^{3^m} = e\},$$

$$\mathcal{K}_{3,3,0} = \text{mod}\{[x, y, z]^3 = e, [x, y]^3 = e\},$$

$$\mathcal{K}_{3,3,3^m} = \text{mod}\{[x, y, z]^3 = e, [x, y]^3 = e, x^{3^m} = e\}, m = 1, 2, \dots$$

Denote by  $\mathcal{B}$  the set of all varieties defined above.

14c<sub>1</sub>). **Lemma 1.** *If a 2-nilpotent Moufang loop  $N$  is finite, then there exists such a variety  $\mathcal{K} \in \mathfrak{B}$  that  $F_3(\mathcal{K}) \in qN$ .*

14c<sub>2</sub>). *Proof.* Since  $N$  is nilpotent we can regard  $N$  as a  $p$ -loop. Let  $\exp(N) = p^m$ . 14c<sub>2</sub>). We consider the following possible cases.

1)  $N$  is non-associative and  $p = 2$ .

2)  $N$  is non-associative and  $p = 3$ .

3)  $N$  is associative and  $p$  is any prime number. Similarly to the previous cases we can show that if in the group  $N$  the identity  $[x, y]^{p^k}$  holds true for a certain natural number  $k$ ,  $1 \leq k \leq m$ , then for  $k = 1$  we have  $F_3(\mathcal{K}_{1,p,p^m} \in q(N)$ .

14c<sub>3</sub>). **Lemma 2.** *If the 2-nilpotent Moufang loop  $N$ , generated by three elements, is infinite, then there exists such a variety  $\mathcal{K} \in \mathfrak{B}$  that  $F_3(\mathcal{K}) \in qN$ .*

*Proof.* Since the loop  $N$  is not finite, then  $\exp(N) = 0$ .

14c<sub>4</sub>). We will consider the following possible cases.

1) Let  $N$  be non-associative, in  $N$  the identity  $(x, y, z)^2 = e$  holds true and  $\exp(lp([u, v]|u, v \in N)) = 2^m s$ , where  $m$  is a non-negative integer and 2 does not divide  $s$ .

2)  $N$  is non-associative, the identities  $(x, y, z)^3 = e$  and  $\exp(lp([u, v]|u, v \in N)) = 3^m s$ , hold true in it, where  $m$  is a non-negative integer and 3 does not divide  $s$ .

3)  $N$  is non-associative, the identities  $(x, y, z)^3 = e$  (respectively,  $(x, y, z)^2 = e$ ) and  $\exp(lp([u, v]|u, v \in N)) = 0$ , hold true in it.

4)  $N$  is non-associative, the identities  $(x, y, z)^2 = e$  and  $(x, y, z)^3 = e$  do not hold true in it.

5)  $N$  is associative and  $\exp(lp([u, v]|u, v \in N)) = 3^m s$  where  $p$  is a prime number and  $p$  does not divide  $s$  and  $m \geq 1$ . In this case we consider in the  $v(N)$ -free group  $F_3(x, y, z)$  the elements  $a = x^s$ ;  $b = y^{p^{m-1}s}$ ,  $c = z^{p^{m-1}s}$  and  $H = lp(a, b, c)$ . Then it is obvious that the loop  $HH$  with exponent zero is non-commutative and the following equalities hold true

$$[a; b]^p = e, \quad [a, c]^p = e, \quad [b, c]^p = e.$$

Then in the non-commutative group  $H$  the identity  $[x, y]^p = e$  is true. Applying the same reasoning as in 1) or 2) we obtain  $F_3(\mathcal{K}_{1,p,0}) \cong H \in q(N)$ .

6)  $N$  is associative and  $\exp(lp([u, v]|u, v \in N)) = 0$ . Similarly to previous cases we can easily deduce that  $F_3(\mathcal{K}_{1,0,0}) \cong H \in q(N)$ .

14c<sub>5</sub>). According the Lemmas 1, 2 and 3 we can formulate the following theorem.

14c<sub>6</sub>). **Theorem 1.** *If  $Q$  is a quasivariety that contains a nilpotent non-associative or non-commutative Moufang loop, then there exists at least one variety  $\mathcal{K} \in \mathfrak{B}$  so that  $F_\infty(\mathcal{K}) \in Q$ .*

14c<sub>7</sub>). From Theorem 1, Lemmas 1 – 3 results the following sentence:

14c<sub>8</sub>). **Theorem 2.** *Cvasivarieties us all non-abelian minimal of the lattice cvasivarieties of 2-nilpotent Moufang loops are: - minimal non-associative quasivarieties of commutative Moufang loops*

$$q(F_\infty(\mathcal{K}_{3,1,0}), \quad q(F_\infty(\mathcal{K}_{3,1,3^m}) \quad (m = 1, 2, \dots));$$

- minimal non-associative and non-commutative quasivarieties of Moufang  $A$ - loops with one proper minimal non-associative sub-quasivariety of commutative Moufang loops and one proper minimal non-commutative sub-quasivariety of groups;

$$q(F_\infty(\mathcal{K}_{3,0,0}), \quad q(F_\infty(\mathcal{K}_{3,3,0}), \quad q(F_\infty(\mathcal{K}_{3,1,3^m}), \quad m = 1, 2, \dots);$$

-minimal non-associative and non-commutative quasivarieties of Moufang loops with the only proper non-commutative subquasivariety of groups

$$q(F_\infty(\mathcal{K}_{2,0,0}), \quad q(F_\infty(\mathcal{K}_{2,2,0}), \quad q(F_\infty(\mathcal{K}_{2,2,3^m}), \quad m = 2, 3, \dots);$$

-minimal non-commutative quasivarieties of groups

$$q(F_\infty(\mathcal{K}_{1,0,0}), \quad q(F_\infty(\mathcal{K}_{1,p,0}) \quad p = 2, 3, \dots);$$

$$q(F_\infty(\mathcal{K}_{1,2,2^m 0}) \quad m = 2, 3, \dots) \quad q(F_\infty(\mathcal{K}_{1,p,p^m 0}) \quad (p \geq 3, m = 2, 3, \dots).$$

14c<sub>9</sub>). Theorems 1 and 2 from items 14c<sub>6</sub>) and 14c<sub>8</sub>) are false, their proofs are "ingenious". It follows from assertions of items 14c<sub>5</sub>), 14c<sub>7</sub>) and the following remarks.

14c<sub>10</sub>). It is necessary to prove the statement from item 14c<sub>2</sub>). For groups it follows from the purpose of given work: describes the smallest quasivarieties and well known assertion: a finite nilpotent group decomposes into direct product of their primary components.

14c<sub>11</sub>). In item 14c<sub>1</sub>) for  $p$  from expression  $\exp(N) = p^n$  are not considered all cases for finite Moufang loop  $N$ :  $p \neq 2; 3$ . Similarly for infinite Moufang loop  $N$  in item 14c<sub>4</sub>).

14c<sub>12</sub>). The assertions of items 14c<sub>10</sub>), 14c<sub>11</sub>) are not connected directly with quasivarieties of groups considered in Theorem 2. For such quasivarieties in items 14c<sub>2</sub>), 14c<sub>4</sub>) the complete, rather short proofs of Lemmas 1, 2 are stated. But these proofs are false, are a complete nonsense. It is not clear what the author V. Ursu has hopes for when publishing such proofs.



14c<sub>13</sub>). Actually the correct proof of statements for groups of Theorem 2 is not simple, involving many mathematics. For example, A. N. Fedorov [19], [21] has found criterion of finiteness of lattice of all subquasivarieties of quasivariety generated by finite 2-nilpotent group. S. A. Shakhova [47] has generalized this criterion for finite generated 2-nilpotent group. In [9] A. I. Budkin has established, that among non-abelian quasivarieties of 2-nilpotent groups without torsion only one quasivariety (particularly, the quasivariety generated by free 2-nilpotent group of rank 2) has finite lattice of quasivarieties.

14c<sub>14</sub>). Clearly that all quasivarieties of 2-nilpotent groups considered in Theorem 2 do not satisfy to criteria listed in item 14c<sub>13</sub>).

14d). For the remaining part of the analyzed work, Theorem 3, Examples 1 – 3, we quote the items 14e), 14e<sub>1</sub>).

14e). **Theorem 3.** *If the alternative ring  $K$  with a unit contains a nilpotent sub-ring  $R$  with index  $n \geq 2$  (i.e. any product of  $n$  factors  $a_1 a_2 \dots a_n = 0$  for any  $a_1, \dots, a_n \in K$ ), then the set  $L$  of all elements of the form  $1 + x$ , where  $x \in R$ , forms a nilpotent Moufang loop of class  $n - 1$ .*

14e<sub>1</sub>). *Proof.* The equality  $(1 + r)(1 - r + r^2 - \dots + (-1)^{n-1} r^{n-1}) = 1$  where  $x \in R$ , shows that any element from the set  $L$  is reversible and, therefore,  $L$  is a Moufang loop.

14e<sub>2</sub>). It is necessary to give reason, to prove the statement of item 14e<sub>2</sub>), see, for example, [48].

14e<sub>3</sub>). The proof of Theorem 3 is very bulky, confusing, it is not clear why designation  $x^* = -x + x'$  is entered. The Theorem 3 is a known fact, the beginning being Mal'cev's result for groups [31]. For commutative Moufang loops the Theorem 3 contained in [22]. In these papers the proofs are rather simple, less computing than in the analyzed work [67]. The proofs become even simpler in Sandu's papers. The very simple proof of Theorem 3 is presented in [35, Lemmas 12, 13]. The form of Theorem 3 was borrowed from this paper. In [38] the proof is less computing as it uses the one-to-one connection between invertible elements and quasiregular elements in alternative algebras [69].

14e<sub>4</sub>). Example 1 uses a deformed variant of the known procedure: externally associated of unit to the given algebra [69, pag. 29]. A 2-nilpotent Moufang loop with exponent 3 is stated, essentially using Theorem 3. In Examples 2, 3 a Moufang loops of varieties  $\mathcal{K}_{2,2,2^2}$ ,  $\mathcal{K}_{2,2,0}$ , a non-commutative and non-associative Moufang loop of variety  $\mathcal{K}_{2,0,0}$  are stated. They will not be analyzed in details, as according to item 14c<sub>14</sub>) these examples do not have any importance for the main results of the given work, Theorems 1, 2.

14f). From the above-stated follows that all results of [66], [67] are

false. These works are plagiarized. In particular, the main notions defined and used in the paper do not belong to author V. Ursu, as follows from papers [66], [67]: they are copied from the works of other authors, without any references.

15a). The work [68] begins with the following citations 15a<sub>1</sub>), 15a<sub>2</sub>).

15a<sub>1</sub>). In [50] is shown that if the periodic part  $P$  of a finitely generated commutative ML  $L$  has exponent  $3^k$ , then the Frattini subloop is  $\phi(L) = L'P^3$ , where  $P^3 = \langle x^3 | x \in P \rangle$ . We will prove that a similar statement is also true for NML.

15a<sub>2</sub>). **Lemma 1.** *If the periodic part  $P$  of the finitely generated NML  $L$  has exponent  $p^k$ , then the Frattini subloop is  $\phi(L) = L'P^p$ ,*

15a<sub>3</sub>). The assertion from item 15a<sub>1</sub>), in such a form is not contained in [50]. A similar statement is presented.

15a<sub>4</sub>). **LEMMA 5.** *If all elements of commutative Moufang loop  $L$  have infinite order or order 3, then  $H' = \Phi(H)$  for every non-cyclic finite generated subloop  $H \subseteq L$ . In particular, if  $L$  is a free commutative Moufang loop with  $n$  generators, then  $L' = \Phi(L)$ .*

15a<sub>5</sub>). According to relation  $L^3L' = \Phi(L)$  from item 8c<sub>3</sub>) it follows that the Lemma 5 in [50] (item 15a<sub>4</sub>)) and the Lemma 1 from item 15a<sub>2</sub>) are false.

15a<sub>6</sub>). Lemma 2 is not proved, as its proof uses essentially the unproved identities (4) – (10), mentioned in item 14b<sub>9</sub>).

15b). Lemma 4 is a particular case for Moufang loops of Theorem 3 in [11] for algebras. The proof of Theorem 3 for groups is contained in Budkin's paper [5], [10]. The proof of Lemma 4 literally coincides with proof of Theorem 3 for algebras (or Budkin's proof), only the homomorphisms, the congruences of algebras are described by normal subloops for loops. [68] does not contain any references to [11] and Budkin's papers. The author V. Ursu did this to assume to himself another person's result, the proof of Lemma 4.

15c). Lemma 3 is plagiarized, as well as Lemma 4. Lemma 3 and its proof literally coincides with the result for groups. The only difference: the expression "nilpotent group" was replaced by expression "nilpotent Moufang loop"; to reasonings for commutators in groups the identical reasonings are added for associators in Moufang loops.

15d). Theorem 1 is the main result of [68]. Let us cite items 15d<sub>1</sub>) – 15d<sub>4</sub>).

15d<sub>1</sub>). **Theorem 1.** *If a Moufang loop  $L$  contains a nilpotent subloop  $H$  that is either non-commutative or non-associative, and all abelian groups*

from  $L$  have ranks bounded by the same number  $r$ , then all quasiidentities that are true in  $L$  do not have a basis of quasiidentities in a finite number of variables.

15d<sub>2</sub>). PROOF. By hypothesis, the Moufang loop  $L$  contains a non-commutative or non-associative subloop  $H$ . By Theorem 1 or 2 (from [67]), in the set  $\mathcal{B}$  there exists a variety  $\mathfrak{N}$  such that all  $\mathfrak{N}$ -free loops are contained in the quasivariety  $q(H)$ .

15d<sub>3</sub>). Let  $t$  be a natural number,  $t > 1$ ,  $m = t(t^2 - 1)/6$  and chose a natural number  $n$  so that  $n > 10m + 1$ .

15d<sub>4</sub>). **Corollary 2.** *The quasivariety which is generated by any finitely generated non-abelian nilpotent Moufang loop  $L$  does not have a finite basis of quasiidentities in a finite number of variables.*

15d<sub>5</sub>). Theorem 1 and its proof are written in the same manner as Lemma 3, see item 15c). Theorem 1 is false and its proof is erroneous, together they make an unsuccessful plagiarism. Naturally, its source is not mentioned. This confirms the statements from this item.

15d<sub>6</sub>). Item 15d<sub>2</sub>) uses the false Theorem 1 or 2, which by item 14c<sub>9</sub>) are false.

15d<sub>7</sub>). The proof of Theorem 1 uses Lemma 1, which by item 15a<sub>5</sub>) is false.

15d<sub>8</sub>). The Theorem 1 and its proof are a mechanical replication of the corresponding results for groups. Item 15d<sub>3</sub>) is necessary for this transfer.

15d<sub>9</sub>). The proof of Theorem 1 literally coincides with the proof for groups. The only distinction between these proofs is the following: the expression "nilpotent group" was replaced by expression "nilpotent Moufang loop"; the expression "commutant" was replaced by expression "associant-commutant". Dear author, for the sake of decency it was necessary to change "commutator  $[x, y]$ " for "associator  $[x, y, z]$ ".

15d<sub>10</sub>). From items 15d<sub>5</sub>) – 15d<sub>8</sub>), 15d<sub>10</sub>) it follows that Theorem 1 from item 15d<sub>1</sub>) is false, its proof is a senseless replication of the group's result on Moufang loops.

15d<sub>11</sub>). Corollary 2 from item 15d<sub>4</sub>) confirms the procedure of mechanical replication. The expression "abelian group" was carelessly transferred on expression "abelian Moufang loop".

15e)). Theorem 1 and Corollaries 2 – 4 are false, as a consequence of Theorem 1.

15f)). From the above-stated it follows that almost all results of work [68] are false, others are plagiarized. The work [68] is a direct continuation of [66], [67].

15g). From items 14f) and 15f)) it follows that the works [66] – [68] do not even represent any mathematical interest, they are anti-scientific.

16a). Finally, we shall consider the work [63], which has a different subject than the other analyzed works. We cite items 16a<sub>1</sub>) – 16a<sub>5</sub>).

16a<sub>1</sub>). Let  $A_i, i \in I$ , be a set of algebraic systems of arbitrary signatures  $\sigma_i, i \in I$ . We complete the signature of every system  $A_i$  with the functional symbols  $p_j, j \in I$ , that correspond to the operations of projections  $p_j^{A_i}, j \in I$ , defined on the Cartesian power  $A_i^I$  with values from  $A_i : p_j^{A_i}(a) = a_j, j \in I$ , for any element  $a = (a_i, i \in I) \in A_i^I$ . If not all systems from this set are algebraic, then we also complete the signature of every system  $A_i$  with the predicative symbol  $e$  that corresponds to the real identical predicate  $e^{A_i}$ , defined on the Cartesian power  $A_i^I$  with real values:  $A_i | = e^{A_i}(a)$  ( $e_i^A(a)$  holds in  $A_i$ ) for any  $a = (a_i, i \in I) \in A_i^I$ . The system we obtain in such a way will be called an *enriched algebraic system* and will be also denoted by  $A_i$ .

The *enriched Cartesian product* of the enriched algebraic systems  $A_i, i \in I$ , is an algebraic system  $\otimes_i A_i$  with the basic set  $A = \prod_{i \in I} A_i$ , which for each family of basic  $n$ -operations ( $f^{A_i}, i \in I$ ) and each family of basic  $m$ -predicates ( $r_i^{A_i}, i \in I$ ) of the enriched systems  $A_i, i \in I$ , has a basic  $n$ -operation  $f^A$  and a basic  $m$ -predicate, defined by

$$f^A(a_1, \dots, a_m) = (f_i^{A_i}(a_1(i), \dots, a_n(i)), i \in I),$$

$$A | = r^A(a_1, \dots, a_m) \Leftrightarrow \forall_{i \in I} A_i | = r_i^{A_i}(a_1(i), \dots, a_m(i)),$$

where  $a_1, a_2, \dots$  are elements from  $A$  and it doesn't have any other basic operations and predicates.

16a<sub>2</sub>). Let now  $Q_i, i \in I$  be a set of classes of algebraic systems. The signatures of these classes may be different. We will define the product of classes  $Q_i, i \in I$ , as the class of algebraic systems, consisting of all isomorphisms of algebraic systems of the form  $\otimes_i A_i$ , where  $A_i \in Q_i, i \in I$ .

16a<sub>3</sub>). **Lemma 1.** *The product of a finite number of filteredly closed classes is a filteredly closed class.*

16a<sub>4</sub>). **Lemma 2.** *The product of a finite number of hereditary classes is a hereditary class.*

16a<sub>5</sub>). **Theorem 4.** *If the classes of algebraic systems  $K_1, \dots, K_n$  are quasi-varieties, then the product  $K_1 \otimes \dots \otimes K_n$  is a quasi-variety.*

16a<sub>6</sub>). The role and importance of the introduced notions of functional symbols  $p_j, j \in I$  and identical predicate  $e^{A_i}$  in definition of enriched algebraic system  $A_i$  from item 16a<sub>1</sub>) is not clear. Trivial is the fact that these

notions do not influence in any way the structure of algebraic system  $A_i$ . Also trivial are these notions for definitions of enriched Cartesian product.

16a<sub>7</sub>). According to item 16a<sub>2</sub>) the signatures of classes  $Q_i, i \in I$  may be different. In such a case Theorem 4 from item 16a<sub>5</sub>) is a nonsense. Even from this follows that every quasivariety is characterized by a set of quasiidentities and that a quasivariety in product of algebraic systems is true iff it is true in ever factor.

16a<sub>8</sub>). The proof of Theorem 4 from item 16a<sub>5</sub>) uses essentially Lemmas 1 and 2 from items 16a<sub>3</sub>) and 16a<sub>4</sub>). It is even useless to comment the proofs of these lemmas. Actually, the correct statements for Lemmas 1,2 are [33, Theorem IV.8.2] and [33, Corollary IV.8.4] respectively.

16a<sub>9</sub>). This obvious example contradicts Theorem 4 (also Theorem 3, 5). According to item 16a<sub>2</sub>) let the classes of quasivarieties (universal classes, varieties)  $K_1, K_2, \dots, K_n$  consist of algebras of arities 1, 2,  $n$  respectively. By Theorem 4 (or Theorem 3 or Theorem 5) the product  $K_1 \otimes K_2 \otimes \dots \otimes K_n$  is a quasi-variety. Question: what arity has this quasi-variety?

16b). It is useless to comment the statement for varieties. They are a nonsense. The correct statements are contained in [33], whence the author of [63] has copied them according to his level of understanding.

16c). Also it is useless and unpleasant to characterize paper [63]. It is straightforward.

Now we present the scheme of conformity of the above-stated items to the papers from the List of References:

1a) – 1o) → [56]; 2a) – 2o) → [16]; 3a) – 3i) → [62]; 4a) – 4c<sub>1</sub>) → [13]; 5a) – 5h) → [61]; 6a) → [60]; 7a) – 7d) → [53]; 8a) – 8j) → [54], [55], [58], [59]; 9a) – 9h<sub>8</sub>) → [58], [59]; 10a) – 10d) → [59]; 11a) – 11n) → [64]; 12a) → [?], [64]; 13a) → [66], [67]; 14a) – 14f) → [66], [67]; 15a) – 15g) → [68], [50], [66], [67]; 16a) – 16b) → [63]; Conclusions → [49] – [57], [17].

## 5 Conclusions

From the above-stated it follows that all of the analyzed works of V. Ursu and A. Covalschi consist only of gross mistakes, in the best case they consist of plagiarism: erroneous mechanical replication of the results for nilpotent groups on centrally nilpotent Moufang loops or  $A$ -loops. These works are not scientific. In more detail.

a). Have no elementary skills of logic thinking, committing gross mis-

takes of computing nature in the proofs.

b). Assumed to himself, changing the terminology, (naturally V. Ursu) various specific features (belonging N. Sandu) of notion of centrally nilpotent Moufang loops or  $A$ -loops. From their works it follows that they have not understood the meaning of the constructive descriptions of notion of centrally nilpotent loop. This notion was introduced by R. Bruck.

c). The basic idea is that all of the analyzed works is a transfer of the results for identities of commutative Moufang loops, separable algebras, for quasivarieties of nilpotent groups on more general algebraical structures. This mechanical replication of results led sometimes to senseless statements. Even worse is the plagiarism of Mal'cev's results.

The above analysis includes all published works of V. Ursu, except for [49], [57], see [17]. The work [57] considerably differs from all works of V. Ursu. This paper is written by skilled expert, based on mathematical logics. The works [13], [61], [62], [16] make the dissertation of A. Covalschi (Kishinev, 2013), the works [49] – [57] make the dissertation of V. Ursu (Novosibirsk, 2000). From the analyzed works it follows that all results of Covalschi's dissertations are erroneous. The same is true for the results from the dissertations belonging to V. Ursu.

At last, naturally there is a question: how could it happen that the analyzed works were published in rather prestigious Scientific journals?

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